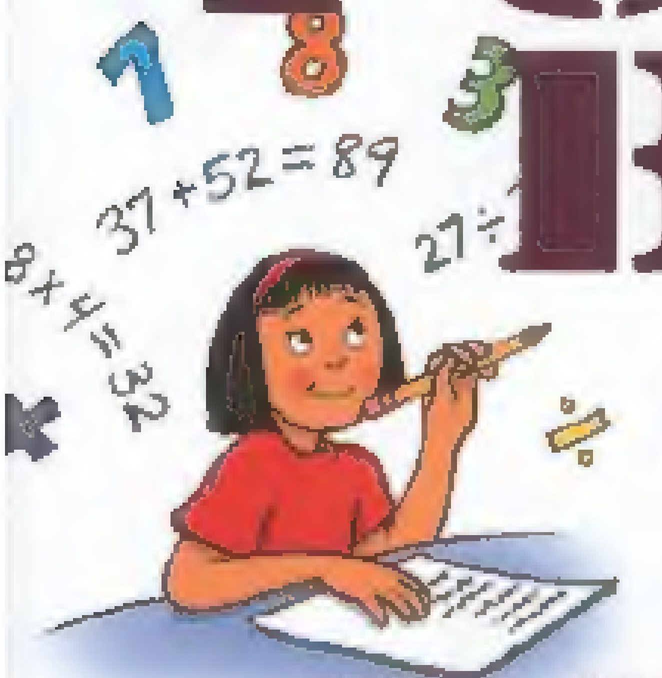
# THE EVERYTHING KIDS'® MATH PUZZLES BOOK


Brain teasers, games, and activities for hours of fun

MEG, GLENN, AND SEAN CLEMENS

$6.95 (CANADA $10.95) JUVENILE

"*The Everything® Kids'* books are fun, challenging, and educational."
—LYNN FAVREAU, RN, HOMESCHOOLER

# STUMP YOUR FRIENDS AND FAMILY!

Who knew that math could be so cool? Crammed with games, puzzles, and trivia, *The Everything® Kids' Math Puzzles Book* puts the fun back into playing with numbers!

If you have any fear of math—or are just tired of sitting in a classroom—*The Everything® Kids' Math Puzzles Book* provides hours of entertainment. You'll get so caught up in the activities, you won't even know you're learning!

**Inside, you'll be able to:**

- **Decode hidden messages using Roman numerals**
- **Connect the dots using simple addition and subtraction**
- **Learn to create magic number squares**
- **Use division to answer musical riddles**
- **Match the profession to numerical license plates**

**Meg Clemens** has a B.A. in mathematics from Wellesley College, an M.S. in mathematics from Wright State University, and an M.S. in teaching from SUNY Potsdam College. She is also a National Board Certified Teacher. Ms. Clemens has been teaching high school math for the past nine years.

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THE EVERYTHING® KIDS'® MATH PUZZLES BOOK

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# The EVERYTHING KIDS'® Math Puzzles Book

Brain teasers, games, and activities for hours of fun

Meg, Glenn, and Sean Clemens

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# Introduction

As math teachers, we often hear these complaints: "I just can't do math" or "Math was never my thing." Sometimes it's students who are frustrated in class; other times it's parents who can't help their kids with math homework. In the latter case, parents may unintentionally pass on their own fear of math to their children.

Although we certainly understand that math comes more easily to some than to others, we believe anyone can "get it" and *should* get it. It's very important that all kids learn math because it will be an essential skill in the twenty-first century. The puzzles in this book will help kids develop skills in arithmetic, geometry, number sense, logical thinking, and problem solving, which form the foundation of mathematical understanding.

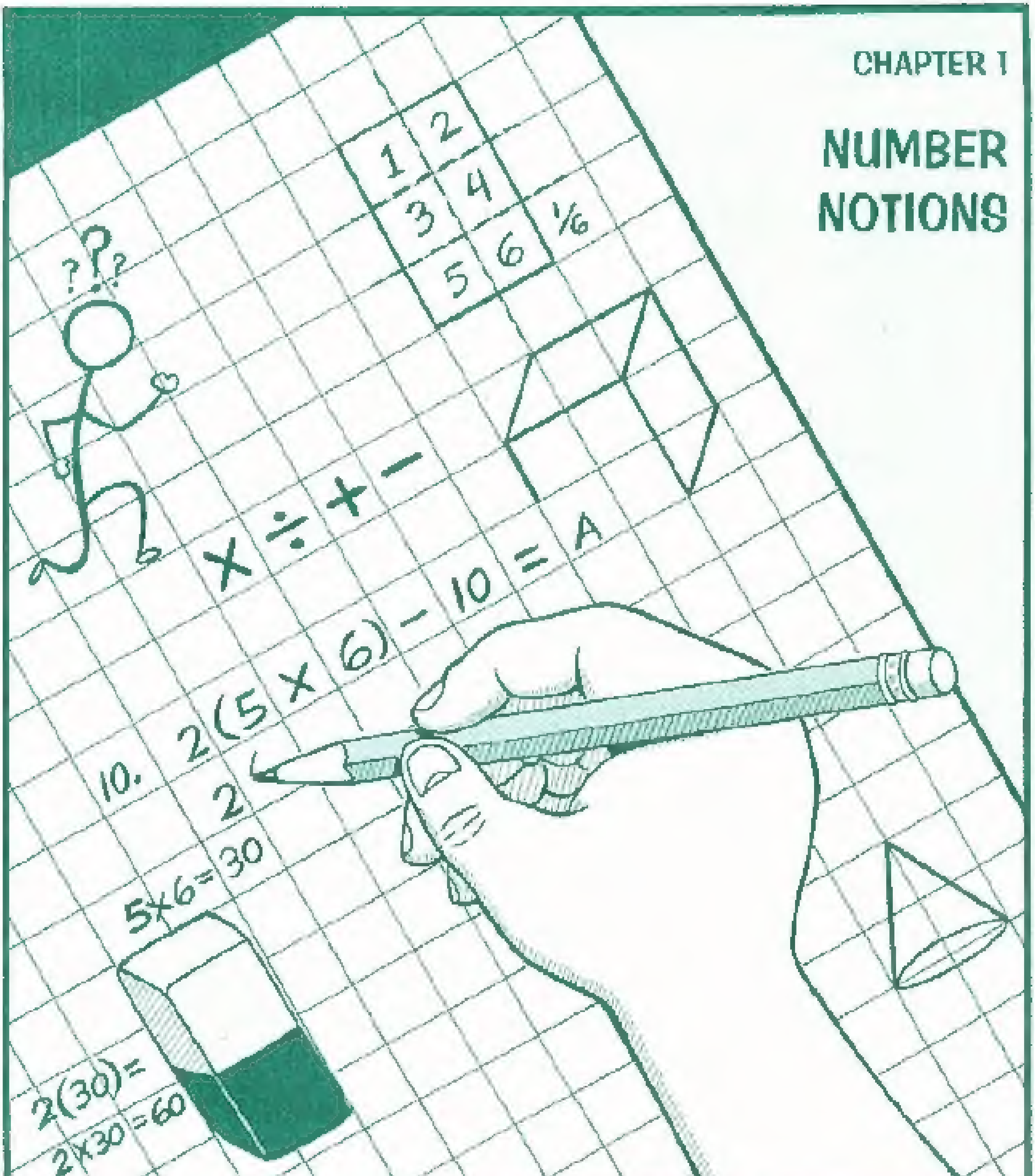
Math is more than just a collection of math facts and vocabulary (although those are important). Math should also be a way of thinking about and solving real problems. You can be an absolute whiz at the multiplication tables, but you won't find them very useful if you are unsure when you

should be multiplying and when you should be dividing. Here is a simple everyday-life problem: There are forty-five fifth-graders going on a class trip to the Museum of Math Puzzles. Parent volunteers with minivans are driving; each van can carry six students. The teachers plan to have enough cookies on the trip for each student to get four. How many cookies do the parents need to prepare and how many vans should they take? If your answer is eight vans and 180 cookies, your field trip will be a success.

Math puzzles are both fun and rewarding—we are confident that any child will enjoy doing the puzzles in this book, and knowing that you've solved a challenging puzzle is definitely rewarding. Beyond simple enjoyment and satisfaction, puzzles also provide wonderful opportunities for learning. Challenging puzzles offer children a chance to practice skills they already know and also to stretch their minds and extend their knowledge by discovering new ideas.

CHAPTER 1

# NUMBER NOTIONS



## A VERY BRIEF HISTORY OF NUMBERS



No one knows who invented numbers, but it's fun to think that some early caveman named Oog did it when he said "uhn," "uhn uhn," and "uuuuuuhhhn," which meant "one," "two," and "lots."

### Tallying

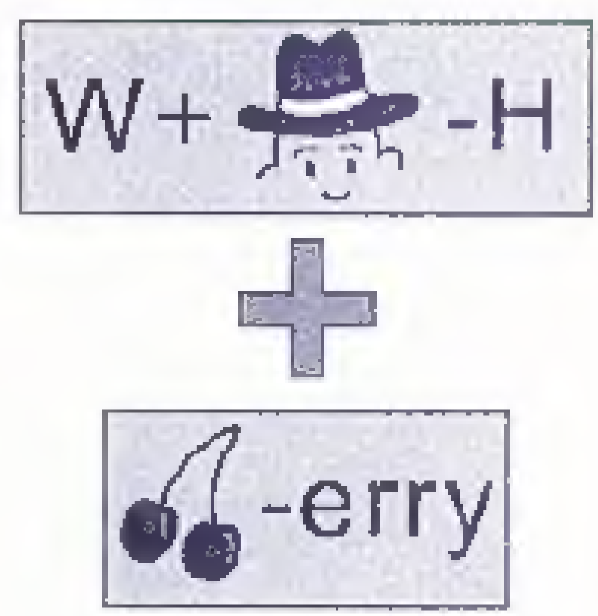
The earliest written numbers were most likely **tallies**—simple lines that each represent one object. You have probably used them yourself to keep score in a game with a friend:


**Counting Sticks**

Tallies were found carved on pieces of animal bone about 50,000 years old. What do you think those people were counting?

### Roman Numerals

The Roman numerals, invented by—yes, you guessed it!—the Romans, served many people over many centuries, and have not been forgotten today. Can you think of how they might be used?




**ROMAN NUMERALS**

| number | Roman numeral |
|---|---|
| 1 | I |
| 5 | V |
| 10 | X |
| 50 | L |
| 100 | C |
| 500 | D |
| 1,000 | M |
| 5,000 | V |
| 10,000 | $\overline{X}$ |

**FUN FACT**

**It's about Time!**

Today, Roman numerals are most often used to show time (see if you have a clock in your house that uses Roman numerals) and dates. Try to find Roman numerals on old buildings and also in movies and videos.

The rules for reading the R[illegible] are [illegible] very simple once you [illegible] hang of it.

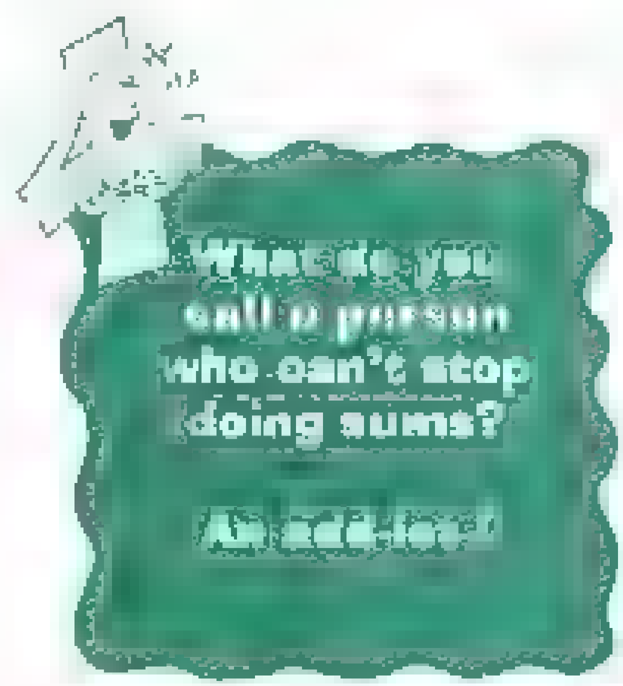
I. When a symbol is followed by a smaller symbol or [illegible] up [illegible]

**For example: VI = 6; CXXIII = 123; DII = 502.**

II. When a symbol or symbols are followed by a larger-value symbol, you subtract their values.

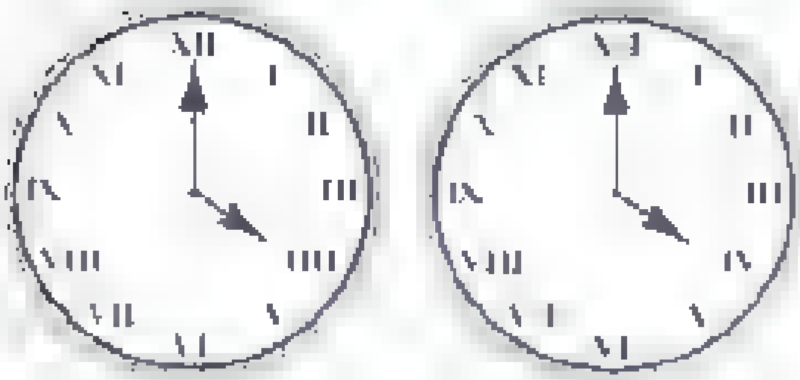
**For example. IV = 4. CMXL = 940; CDII = 402.**

So, the numbers for 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 are I, II, III, IV, V, VI, VII, VIII, IX, and X. Are you beginning to catch on? If so, see if you can handle the following p[illegible]



## Figure This Out

**Which face would you see on a grandfather clock?**



Grandfather clocks use Roman numerals for each hour, but with a surprise: Four o'clock is represented with IIII instead of IV. See if you can find a watch with Roman numerals on its face and check out four o'clock.

> **Try This**
>
> ### Read All about It!
>
> Want to practice your Roman numerals? How about counting a thousand squealing pigs? Just grab *Roman Numerals I to MM* by Arthur Geisert (Houghton Mifflin Company, 1996) and let the fun begin. We bet you never knew there could be so many pigs in one picture!

## When in Rome

**Question:**
**What would you use to count organic apples?**

First, take a look at these Roman numerals and see if you can figure out what numbers they represent.

| a | b | e | | m |
|---|---|---|---|---|
| XIV | XXIX | VIII | LXI | CDXI |

| n | r | s | t | u |
|---|---|---|---|---|
| CXLIX | MDCVI | DCCLI | MMDCLX | CMXXII |

Now place the letter for each Roman numeral under the corresponding Arabic number to find the answer to the puzzle.

**Answer:**

| 149 | 14 | 2660 | 922 | 1606 | 14 | 61 |
|---|---|---|---|---|---|---|
| | | | | | | |

| 149 | 922 | 411 | 29 | 8 | 1606 | 751 |
|---|---|---|---|---|---|---|
| | | | | | | |

Adding and subtracting with Roman numerals is not easy, and multiplication and division are nearly impossible.

XLIII + XXIV = ______________ (LXVII)
XCII − XXVI = ______________ (LXVI)
XV × IV = ______________ (LX)
XVIII ÷ III = ______________ (VI)

## Arabic Numerals

[illegible] numerals in favor of the Arabic system, which relies on ten digits [illegible]

### WORDS 2 KNOW

*digit.* A number—but also a word for "finger." Coincidence? We think not. It's very likely that the first people to start counting used their fingers, just as little kids continue to do today.

## Hidden Numbers

Each of the following sentences has at least one hidden Arabic number. Circle as many as you can find.

1. I love my computer—when it works!
2. Beth reeked of smoke after sitting by the campfire.
3. My mother likes to weigh tomatoes on every scale in the store.
4. Annie was even early for school last week!
5. We can stuff our dirty backpacks in your tent.
6. We like the mirrored maze room at the fun park.

### Practice Your Digits

[illegible] from START [illegible] END by [illegible] [illegible] are either [illegible] up [illegible] [illegible] [illegible] or right [illegible] the number [illegible] [illegible] you are on. For example [illegible] three squares [illegible] or [illegible] three spaces up, down, left, or right.

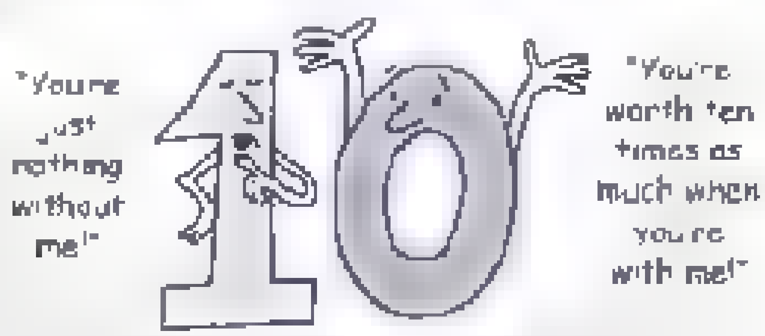
| | | | | |
|---|---|---|---|---|
| 3 | 3 | 4 | 2 | 3 |
| 4 | 1 | 1 | 2 | 2 |
| 2 | 4 | 3 | 1 | 4 |
| 2 | 3 | 1 | 4 | 3 |
| 4 | 2 | 1 | 2 | E |

| | | | | | |
|---|---|---|---|---|---|
| 4 | 2 | 1 | 2 | 3 | 4 |
| 3 | 3 | 4 | 2 | 3 | 4 |
| 4 | 2 | 4 | 2 | 4 | 3 |
| 3 | 2 | 1 | 3 | 2 | 4 |
| 2 | 2 | 4 | 2 | 3 | 4 |
| 4 | 1 | 4 | 1 | 2 | E |

## THE IMPORTANCE OF ZERO

How important is "nothing"? Sometimes [illegible] very important. But we humans don't always understand what exactly "nothing" is.

We need zero for [illegible] reasons. For one thing, how can you [illegible] Furthermore, we need zero [illegible] Otherwise, [illegible] [illegible] the difference between [illegible] and [illegible] [illegible]




### Counting Sheep

Numbers were invented to count things. For example, an early herdsman might have wanted to know how many sheep he had. If he had no sheep, he wouldn't have wasted his time counting them, so he didn't need a number for them—that's why people didn't need zero for a long time!

**What's 10 + 8 - 3 + 12 - 7 - 5 - 15 ?**

[illegible]

## ALL YOU NEED IS 0 AND 1

| BINARY SYSTEM | |
|---|---|
| binary number | Arabic numeral |
| 0 | 0 |
| 1 | 1 |
| 10 | 2 |
| 11 | 3 |
| 100 | 4 |
| 101 | 5 |
| 110 | 6 |
| 111 | 7 |
| 1000 | 8 |
| 1001 | 9 |
| 1010 | 10 |

What does math really fast and always gets the right answer? No, it's not that smart kid at school—it's the computer.

Computers don't have any fingers [illegible]. Instead, early computers had simple circuits that act sort of like ordinary light switches—they turn on (1) and off (0). So computers do all their math by just using zeros and ones. This is known as a binary system.

How does the binary system work? Well, it's all about the number placement and the powers of 2. Each place $n$ in the sequence that is "turned on" (marked with 1) stands for a 2 raised to the power $n$.

1 ($2^0$) 2 ($2^1$) 4 ($2^2$) 8 ($2^3$) 16 ($2^4$) 32 ($2^5$) 64 ($2^6$) 128 ($2^7$) . . .

So, a binary number 110 is actually $2^2 + 2^1 + 0$, or $4 + 2 = 6$.

### On or Off?

What time is it when the math teacher goes to the dentist? Convert the binary numbers to find out.

**Number Decoder**

| | |
|---|---|
| H | 6 |
| I | 3 |
| O | 8 |
| R | 2 |
| T | 7 |
| Y | 10 |

111 1000 1000 111 110 -
__ __ __ __ __ -

111 110 11 10 111 1010
__ __ __ __ __ __

## A NUMBER WITH A NAME

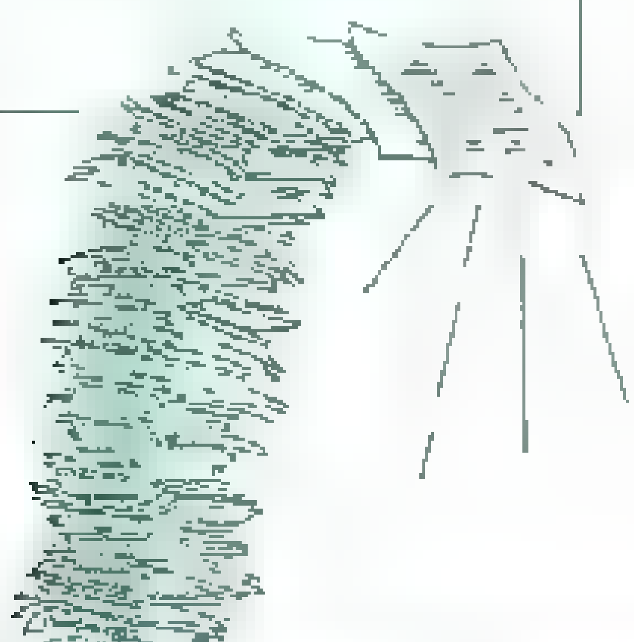
Now you see that we just can't [illegible] zero! But there is [illegible] other special number that is very important in mathematics. That number is π [illegible] pronounced [illegible] and it represents the ra[illegible] [illegible]ence of a circle to its diameter: what you get if you divide the [illegible] around the [illegible] (circumference) by the distance [illegible] diameter.

### WORDS 2 KNOW

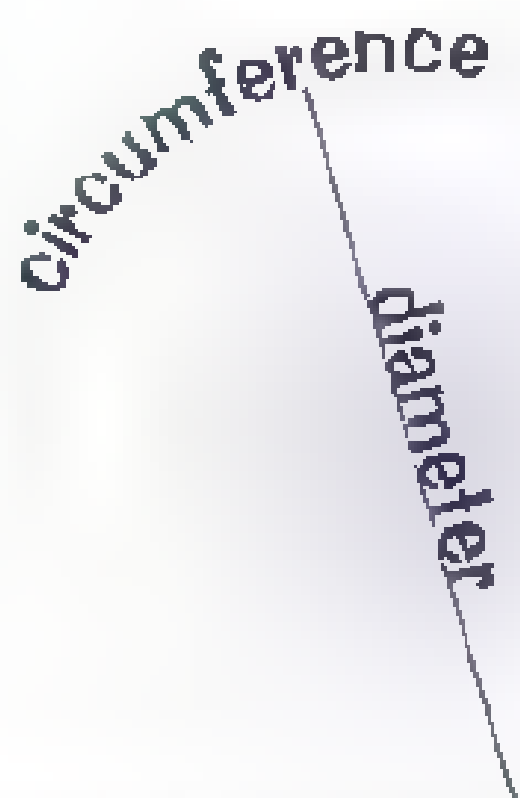
***mathematics:*** Based on the Greek word *manthanein* (to learn), mathematics measures and describes the world with numbers and symbols.

### How Close Were They?

In 1957 Pegasus computer calculated π to 7,840 decimal places. And in 1967, CDC 6600 broke a record by giving us π to 500,000 decimal places. Can you imagine how much paper would be necessary just to print out this number?

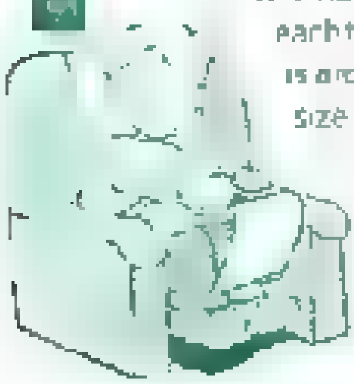


What's surprising to many people is that the answer turns out to be the same for any circle, no matter how big or small it is.



## A Slice of π

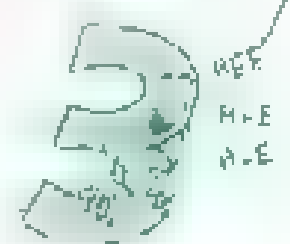
You can approximate the value of π with some string, a ruler or yardstick, and a calculator. Search your home for circular objects of different sizes: a cup, a large jar, a bicycle tire. For each item, find its circumference by wrapping the string around it and then measuring the string. Then find the diameter by measuring across the widest part. Then divide the circumference by the diameter. Because no one can measure perfectly, you will not get the exact same answer each time. But you should find that π is around 3.1 to 3.2 no matter what size circle you measure.



FUN FACT

### The European Way

In the European decimal notation, decimals and commas are switched. A decimal is used to separate groups of thousands and a comma is used between the units' place and the tenths' place. For example, the number 1,234.56 is written as 1.234,56. This has confused plenty of Americans in Paris.



People knew about π for a long time, but they had trouble trying to figure out its exact value. Here are a few examples.

- **Ancient Babylonians estimated π to equal 3.**
- **Ancient Egyptians thought it to be 3.1605.**
- **An ancient Greek named Archimedes estimated π to be between $3\frac{10}{71}$ and $3\frac{1}{7}$.**
- **In China 1,600 years ago, Tsu Ch'ung Chi measured it to be $\frac{355}{113}$.**
- **A great Arab mathematician, al-Khwarizmi, calculated π to be 3.1416.**

Consequently, mathematicians continued getting more precise until humans got some help from computers.

## Making Sense of the Irrationa

The value of π has fascinated people for centuries. Mathematicians and ordinary people have spent years looking for patterns in its digits. Do some digits appear more frequently than others? **Count up how many times each digit appears below and see if you can detect a pattern.**

Here are the first 201 digits of π

3 415926535....8979323846 ....2643383279

5028841971 .6939937510. ....5820974944

5923078164 . 0628620899 .. 8628034825

3421170679 . 8214808651 ......3282306647

0938446095 . 5058223172 ... 5359408128

4811174502.... ...8410270193 ......8521105559

6446229489 ....5493038196

### WORDS KNOW

**irrational:** To many people **irrational** means foolish or even crazy, and maybe that's how you are feeling about π about now! Well, mathematicians agree, but for a different reason. In mathematics, irrational numbers are those that cannot be expressed as a fraction—they go on forever and ever with no repeating pattern.

| digit | tallies | total |
|---|---|---|
| 0 | | |
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |
| 8 | | |
| 9 | | |

**Now, you can graph the results on the facing page to see if a pattern emerges.**


DIGIT VALUE
0 1 2 3 4 5 6 7 8 9
NUMBER OF DIGITS
25 24 23 22 21 20 19 18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1

# Let's Get Packing

[illegible] what Kayle and Dustin had to do to get ready for their hiking trip.

| | | | | | |
|---|---|---|---|---|---|
| Cal | and | bug | to | extra | black |
| bottles, | Kelly | and | flashlight | Find | snacks, |
| Buy | Pack | bird | water | directions | spray. |
| the | Fi[illegible] | for | and | ponchos | books |
| Short | binoculars | Check | sun | batteries. | and |
| chocolate! | park | make | socks | State | get |

1 __________________________________________

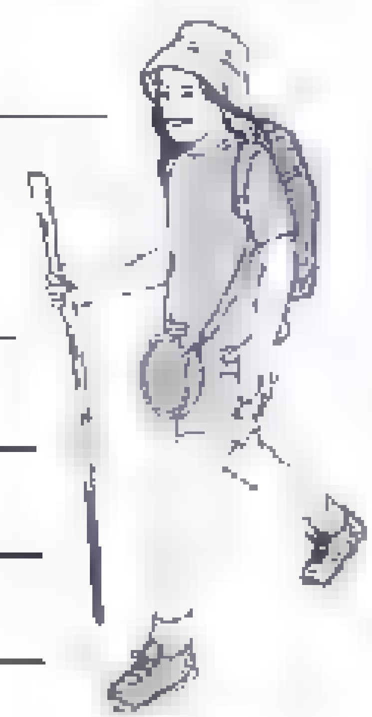
2 __________________________________________

3 __________________________________________

4 __________________________________________

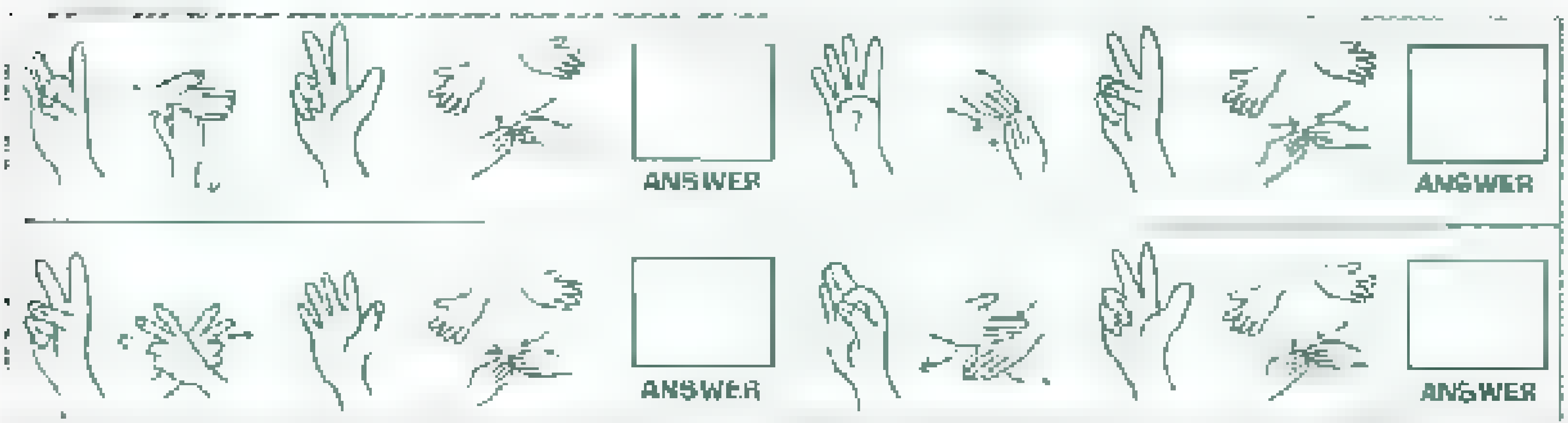
5 __________________________________________

6 __________________________________________



# See What I Mean?

... calculator ... American Sign Language can use hand signals. Stu... each sign on ... page equat... See if you can sign the answer, too.

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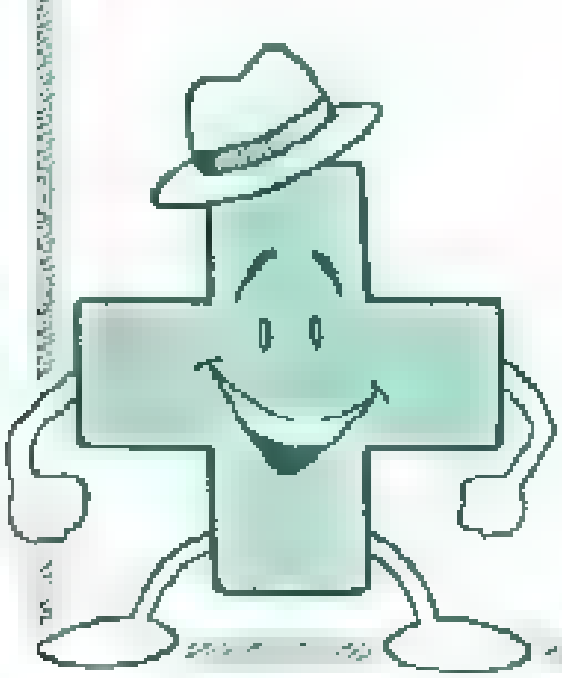
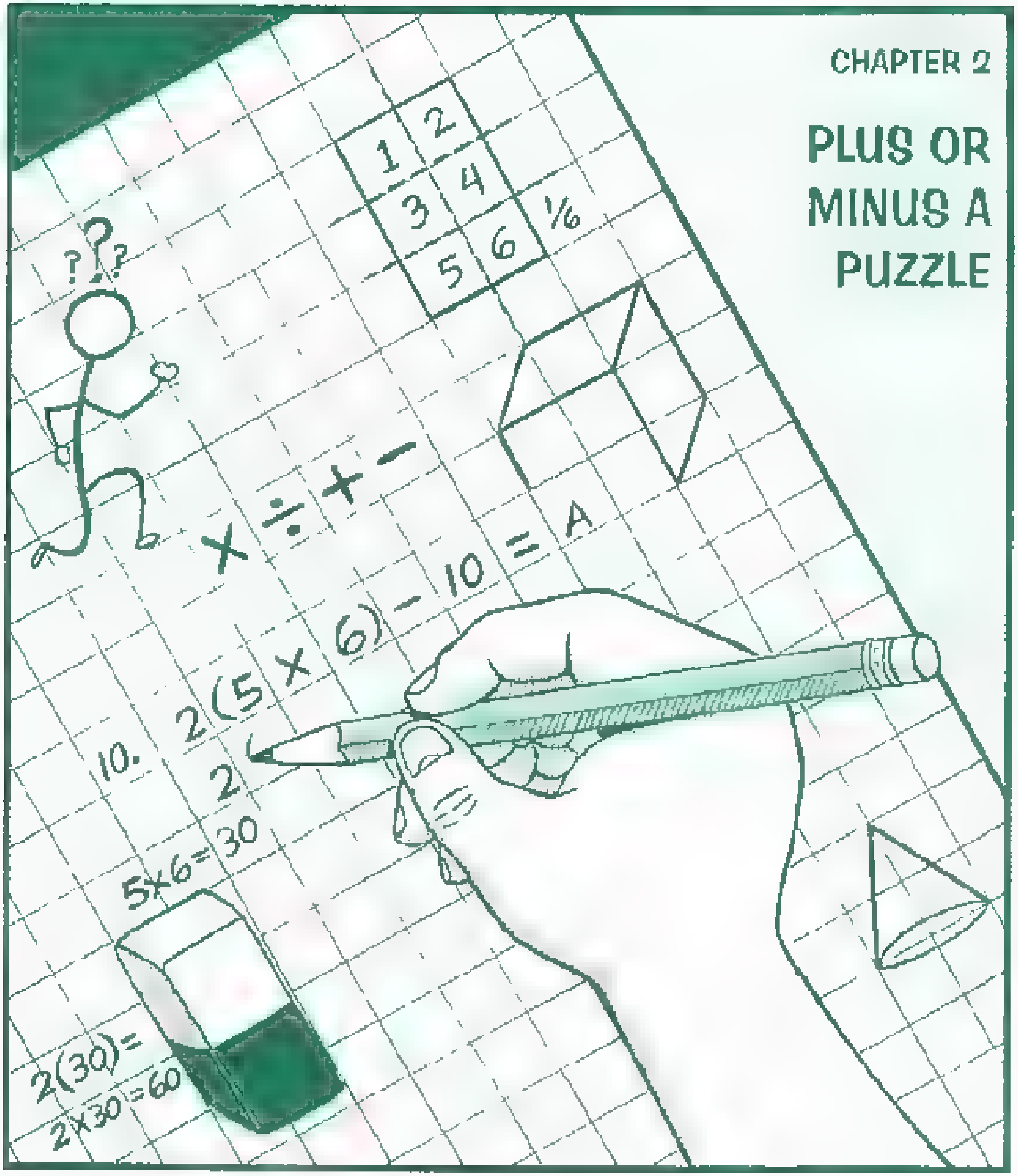


CHAPTER 2

# PLUS OR MINUS A PUZZLE



## ARITHMETIC ACTIVITIES

Arithmetic is doing [illegible] numbers, especially addition, subtraction, multiplication and division. If you already know most of your basic arithmetic [illegible] the following activities and puzzles.

> **WORDS 2 KNOW**
>
> ***calculate:*** This term originated from the Greek word *kalyx*—pebble or small stone—because a long time ago, Greeks used small stones to do simple calculations.

### Clock Math

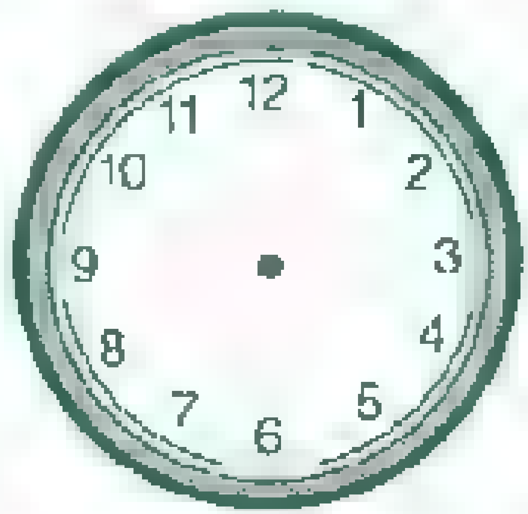
A[illegible] is full of [illegible]. [illegible]? Try this question on a friend. When does **10 + 4 = 2?** [illegible] sensible and important answer: **When it is 4 hours after ten o'clock.**

A[illegible] adding and subtracting regular [illegible]bers. Use the [illegible] to solve the following problems. Then use the [illegible] to the [illegible]

**What time did the math teacher go to the dentist?**

Decoder

| | |
|---|---|
| A = 1 | O = 7 |
| C = 2 | R = 8 |
| H = 3 | S = 9 |
| I = 4 | T = 10 |
| L = 5 | U = 11 |
| M = 6 | Y = 12 |



| | |
|---|---|
| 10 + 12 = ____ | 11 + 4 = ____ |
| 3 + 4 = ____ | 8 + 3 = ____ |
| 10 + 9 = ____ | 11 [illegible] 3 = ____ |
| 1 [illegible] 3 = ____ | 11 - 1 = ____ |
| 5 - 2 = ____ | 12 + 12 = ____ |

## Calendar Math

Here is another practical arithmetic problem. When does 5 + 3 = 1? Three days after the fifth day of the week: from Thursday (5) to Sunday (1).

It doesn't really make sense to say "Thursday + 3 = Sunday," but if you give each day of the week a number, then you can do calendar arithmetic exactly as you did clock arithmetic. The trick is: if your result is over 7, just subtract it from the total. In our example, 3 + 5 = 8; 8 − 7 = 1. Now, what day is 12 days after a Wednesday? 4 + 12 = 16; 16 − 7 = 9; 9 − 7 = 2. This means that 12 days after Wednesday is a Monday—two weeks from then.

Sunday = 1
Monday = 2
Tuesday = 3
Wednesday = 4
Thursday = 5
Friday = 6
Saturday = 7

"The different branches of Arithmetic—Ambition, Distraction, Uglification, and Derision."

—Lewis Carroll, Through the Looking Glass

### When Numbers Don't Obey

Why don't hours of the day and days of the week work the same way as "normal" numbers? Maybe it has to do with limits. Normally, numbers go up as far as infinity, so you never have to start over. With defined terms like the day (which can never have more than twenty-four hours) or the week (which can never have more than seven days), you can't go on forever and therefore need to start over, which messes up the calculation. Can you think of any other instances when numbers don't behave normally?

## NUMBERS WITH DIRECTION

Recall that numbers were originally used to count things, like sheep, and it took centuries before a symbol was invented to represent zero. **Are there any numbers that are <u>less</u> than zero?**

Today, most people have heard of negative numbers and know that they are not at all imaginary and are really [illegible]ful. Think of where you may have heard of numbers like [illegible]5 and [illegible]10. Did you think of a thermometer? Negative numbers represent temperatures below 0 degrees (those are the cold days, whether you use a Fahrenheit or Celsius [illegible]).

For every "positive" number there is its twin "negative," and vice versa. These pairs are called "opposites": 3 and [illegible], 12 and [illegible]12, 135,000,789 and [illegible]135,000,789.

**Using a simple number substitution (A = 1, B = 2, C = 3, and so on), figure out the coded word below to get the name of a useful math tool that will help to show you negative numbers and how they work.**

14 21 13 2 5 18

12 9 14 5

**Q: How many times can you subtract 7 from 83, and what is left afterward?**

**A: I can subtract it as many times as I want—it leaves 76 every time!**

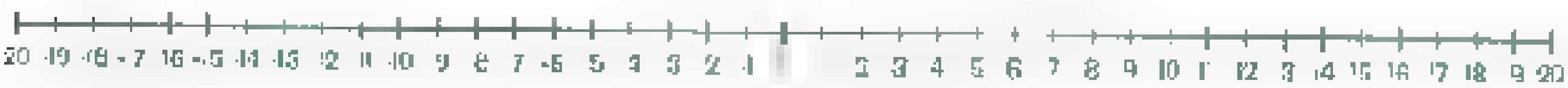
### Adding Signed Numbers

**Why would we want negative numbers?** One reason is that mathematicians don't like problems that have no answer. Everyone knows that we can add any two numbers, say 5 + 3. Most people also know that in addition, the order of the numbers doesn't matter. The answers [illegible] the same.

Now [illegible] try subtraction. 5 [illegible] 3 is no problem: everybody gets 2. But what if we switch the numbers? What is 3 [illegible] 5? Is it still 2? Think of what subtraction means. If you have five pieces of candy and give three to your little brother (very generous of you!), then you [illegible] pieces left. But if you have three pieces of

candy and give five to your little brother ... hey, wait a minute, you can't do that! And if you could, you sure wouldn't have two pieces left. So at least with candy, 3 – 5 doesn't make sense; there is no answer.

Mathematicians really hate that. If you can do 5 – 3, you ought to be able to do 3 – 5. So what is the answer? To find out, we use a **number line.** Notice that if you pick any number on the number line, all the numbers to its left are less than it, while all the numbers to its right are greater.




**Adding 5 + 3 on the number line is easy.** You start at the 5, and then count three more spaces to the right and see where you end up (which is always on 8, since 5 + 3 = 8). Note that starting at the 3 and counting five spaces to the right gets you to the same place (3 + 5 = 8). Subtracting is just as easy. To do 5 – 3, you start at the 5 and count three spaces to the left.

**Now, what about 3 – 5?** After you pass 0, you get to –1 (one less than zero), –2 (two less than zero), –3 (three less than zero), and so on. We use a negative sign in front of the number to show numbers that are to the left of zero on the number line. If you move five spaces to the left of 3, you get to –2. So 3 – 5 = –2. Now, what about 2 – 7? If you got –5, you are on the right track.

FUN FACT

**Hard to Believe!**

Even after mathematicians began thinking about negative numbers—numbers that are less than zero, many of them called such numbers "fictitious," "imaginary," or "useless."

The Bottom Line

moving left

moving right

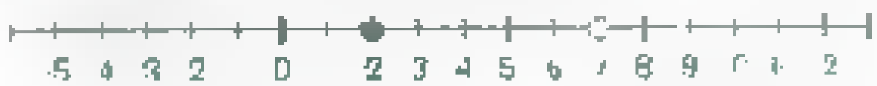
**But wait—there's more!** You can actually add and subtract using negative numbers as part of the problem, instead of just the answer. Try these:

$$7 + (-5) = \_\_$$

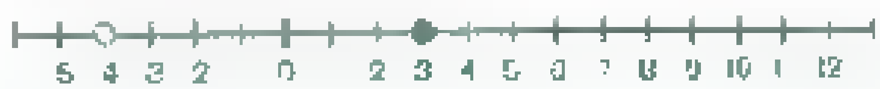
$$-4 - (-7) = \_\_$$

Did you figure out how to do it, or does it look totally impossible? Once you know the rules, it's really easy. Just keep thinking about the number line. If you see a +, you are counting to the right. If you see a –, you need to count to the left.

Let's do 7 + (–5) first. This is an addition problem, which means you need to count from 7 to the right. *However*, because the second number is negative, you actually have to switch directions and count five spaces to the left. That means:



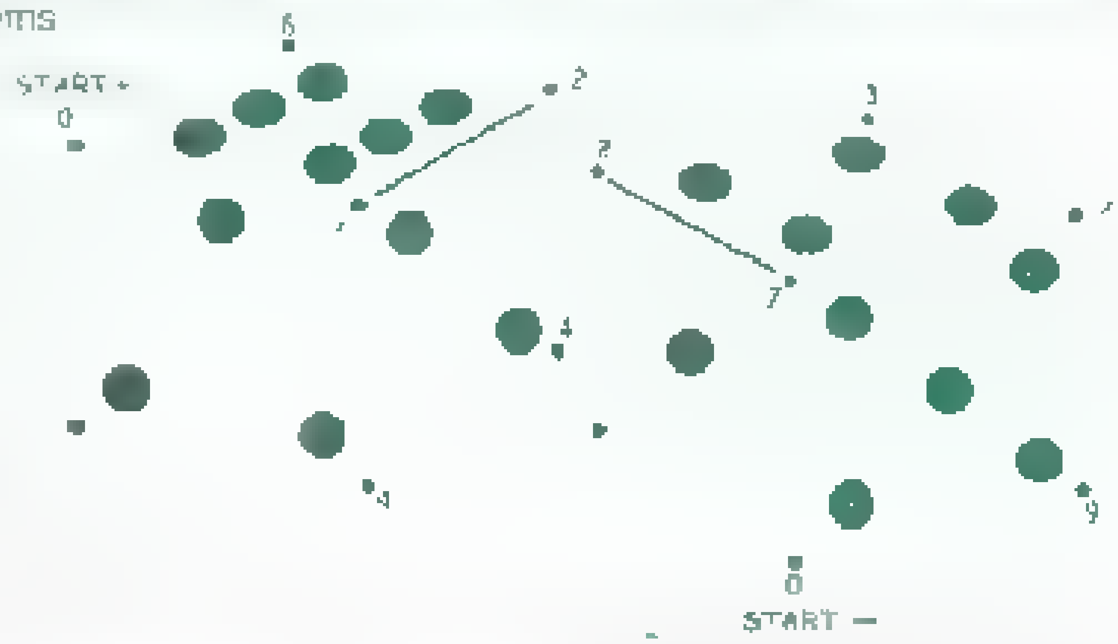

The same approach should work for –4 – (–7). Start at –4. Subtraction means moving to the left, but since the second number is negative, you do the opposite and move right seven spaces, so –4 – (–7) = 3.




## Connect the Dots

**Solve the following addition and subtraction problems using the number line.** [illegible] **START +** [illegible] problems. Then [illegible] **START –** and do the subtraction problems.

| Dot Grid | Addition | Subtraction |
|---|---|---|
| A | -3 + -5 = | 3 - -5 = |
| B | 4 + 2 = | 4 8 = |
| C | 1 + 5 = | 5 2 |
| D. | 4 + -8 = | -4 - -2 = |
| E | 7 + 2 = | 7 -2 = |
| F | 3 + 3 = | -2 - -2 |
| G | 3 + 4 = | 3 4 = |
| H. | 2 + 2 = | -1 1 = |



## MAGIC SQUARES

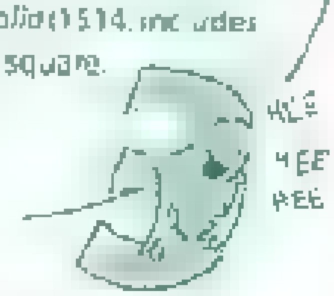
Magic squares have been p[illegible] puzzles for over [illegible] years. [illegible] once were thought to have magical powers. Follow these simple rules [illegible] your own magic square:

1. [illegible]se each number [illegible]
2. Ev[illegible] column [illegible] and diag[illegible] must add up to the same [illegible].

### FUN FACT

**Historical Squares**

Over 3,000 years ago, ancient Chinese included magic squares in their mystical writings. Magic squares also appeared in art. For instance, Albrecht Durer's famous engraving *of Melancholia* (1514) includes a picture of a magic square.




For this first magic square, use the numbers from 1 to 9. HINT: Each column, row, and diagonal adds up to 15.

| | | |
|---|---|---|
| | | |
| | 5 | |
| | | |

Try again, but this time use only the EVEN numbers from 2 to 18 (2, 4, 6, and so forth). HINT: Each column, row, and diagonal adds up to 30.

| | | |
|---|---|---|
| | | |
| | 10 | |
| | | |

Try again, but this time use only the ODD numbers from 1 to 17 (1, 3, 5, and so forth). HINT: Each column, row, and diagonal adds up to 27.

| | | |
|---|---|---|
| | | |
| | 9 | |
| | | |

Now, let's make things more difficult. In the following magic square, some of the numbers are negative—you need to use every number from –4 to 4. HINT: Each column, row, and diagonal adds up to 0.

| | | |
|---|---|---|
| | | |
| | 0 | |
| | | |

Here is another magic square. This time, use the numbers from 3 to 5. HINT: Each column, row, and diagonal adds up to 3.

| | | |
|---|---|---|
| | | |
| | 1 | |
| | | |

So, now you think you are a magic square pro? Not so fast! Let's make the magic square larger and see if you can still solve the puzzle. For the following square, use the numbers from 1 to 16. HINT: Each column, row, and diagonal adds up to 34.

| | | | |
|---|---|---|---|
| 16 | | | 13 |
| | | | |
| | | | |
| 4 | | | 1 |

## How Big Can You Go?

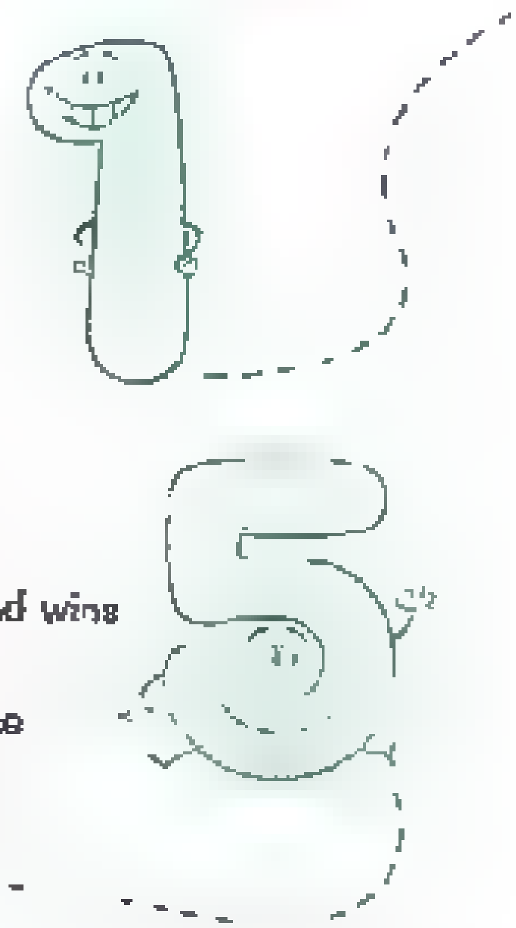
## GAME OF 15

Here is another addictive game you can play! All you need is another player, a piece of paper, and a pen.

First, write down the numbers 1 through 9 on a sheet of paper like this:

1 2 3 4 5 6 7 8 9

Then, you and another player take turns selecting numbers. When a number is selected, the player who chose it crosses it off the list and it can no longer be picked. The first person to get *any* 3 numbers that add up to exactly 15 wins.



**For example:**

**Player #1 picks 5**
**Player #2 picks 8**
**Player #1 picks 6**
**Player #2 picks 3**
**Player #1 picks 1**
**Player #2 picks 7**
**Player #1 picks 9—and wins**

**Player #1 wins because**
**1 + 5 + 9 = 15**

### Try Playing Game of 15 Yourself!

To distinguish who picked which numbers, Player #1 can cross off the numbers by marking an X over it and Player #2 can circle the numbers he or she picks.

1 2 3 4 5 6 7 8 9

1 2 3 4 5 6 7 8 9

1 2 3 4 5 6 7 8 9

## SOLVE A CROSS-NUMBER PUZZLE

You have probably seen crossword puzzles, and it's possible that you have even solved one or two yourself. But what about cross-number puzzles? Does it sound like something you'd like to try? The cross-number puzzle in this chapter is special. In order to solve it, you have to read the following story for clues, so pay attention!

### The Camping Trip

There was an old lady who lived in a shoe. She had so many children she didn't know what to do.

Well, one day, she just couldn't stand it any more. Jamie was fighting with Orville. Wilbur [illegible] a snack. Oscar and Ophelia were [illegible] [illegible] pictures on the walls, and Hortense was crying because D[illegible] had stolen her *Math Puzzles* book. And it was just seven o'clock in the morning.

"Enough already," shouted their mother. "I need some peace and quiet. A[illegible] play outside for the rest of the day. Better yet canoe over [illegible] Mystery Island and play there. Even better, make it a camping trip. You everybody go camping on Mystery Island. Don't come back until Christmas."

At first this seemed rather harsh but then everybody remembered [illegible] Christmas was just two days away. This got the children, who had been quiet for just a moment, all excited again. "Out," yelled the mother.

The children immediately began preparations. With three [illegible] [illegible] they needed [illegible] dozen candles [illegible] [illegible] for each canoe. Each kid grabbed [illegible] underwear, two extra pairs of so[illegible] exactly four [illegible] of food and fifteen pounds of equipment: towels, cups, silverware, sleeping bags, flashlights, handheld video games. Hortense took along her *Math Puzzles* book.

A[illegible] eight o'c[illegible] they p[illegible] the canoes and [illegible] Mystery Island. The trip [illegible] three quarters of an hour and [illegible] they had almost [illegible] the island, Jam[illegible] the edge of [illegible] number 29."

[illegible] asked Gertrude from another [illegible] "You're [illegible] upside down."

"I am?" asked Jamie, and tried to twist around to see it right and [illegible] shore. "This water is FREEZING!"

[illegible] the tents. [illegible] the kids wanted to sleep three to a tent, but they didn't have eno[illegible] so they crowded in four to a tent. Five of the tents were just enough [illegible] boys, the others were for the girls. Unfortunately, no one had remembered to bring tent pegs, so the rest of the morning was spent scrounging up pieces of wood until there was enough for twelve pegs for each tent. The kids then ate a hearty lunch of chocolate-chip cookies.

The next order of business was to build a fire, which wasn't easy because so much of the available wood had been used for tent pegs. Things weren't looking too hot until Rupert [illegible] Rupert [illegible] that was [illegible]

The [illegible] exploring the island, playing games [illegible] get for Christmas. Jam[illegible] her [illegible] the fire. [illegible] wind blew her cards around and the Seven of Hearts landed in the fire

Dinner was more cookies, toasted over the fire this time. After that, the children all sat around telling ghost stories. This was great fun until Mabel, with big eyes, said quietly, "Uh, guys, I think I hear something creeping around in the woods!"

Instantly, everyone was quiet and all ears were listening for noises. After a few minutes, they all heard a distinct "whooo [illegible]" [illegible] sleeping bags and didn't come out until morning.

The next day Lysander got a length of rope and tied knots in it every seven feet to make it easier to hold on to. He ended up with nine knots, including one at each end. The children spent the morning making up different teams and waging tug-of-wars with Lysander's rope until it broke. Later in the day some of them held a fishing contest. Maxwell won by catching the most fish. Penelope caught half as many as Maxwell. Guinevere caught a third as many as Penelope did, and Baxter caught one-quarter as many as Guinevere. Baxter's fish was so small that he let it go. No one else caught anything. That night, they had a fish feast.

The next morning, all the children were up early and eager to get [illegible] were [illegible] to do [illegible] together and to [illegible] a big [illegible] [illegible] contest. The [illegible] was [illegible] the kids tried [illegible] see them back [illegible] Christmas [illegible] spent on the lake that Christmas morning.

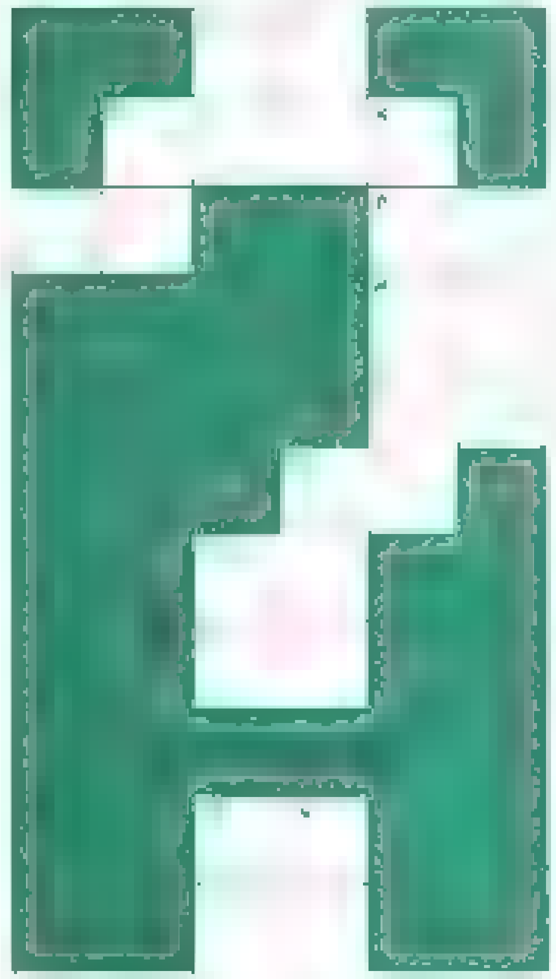
## Now you can try to solve this crossword puzzle using the clues you read in the story . . .

**Across**

1. Number of campers on the trip
3. Date on which they returned home
5. Total length of the supercanoe
6. Total number of canoes plus tents
8. How many minutes it took to get home
9. Total number of hours the trip lasted
10. Number of paddles on way home
11. Total number of fish caught during the contest
12. Length of Lysander's rope (before it broke)
13. Total weight of food and equipment in each canoe (pounds)
15. Number of hours from noon to midnight

**Down**

1. Water temperature, according to Jamie
2. Jamie's canoe number
3. Number of girls on the trip
4. JAMIE on the telephone
7. Total number of tent pegs needed
10. Total number of socks on the trip
11. Number of minutes it took to get to the island
13. Number of cards left in Jamie's deck
14. Number of ears that listened for ghosts



## A Puzzle of Your Own

Did you like the story and the cross-number puzzle? Maybe you can create one for your friends or family to solve. You can be as inventive as you like—to provide clues, make up a story, provide word problems, or simply give clues that the puzzle solver is likely to know.

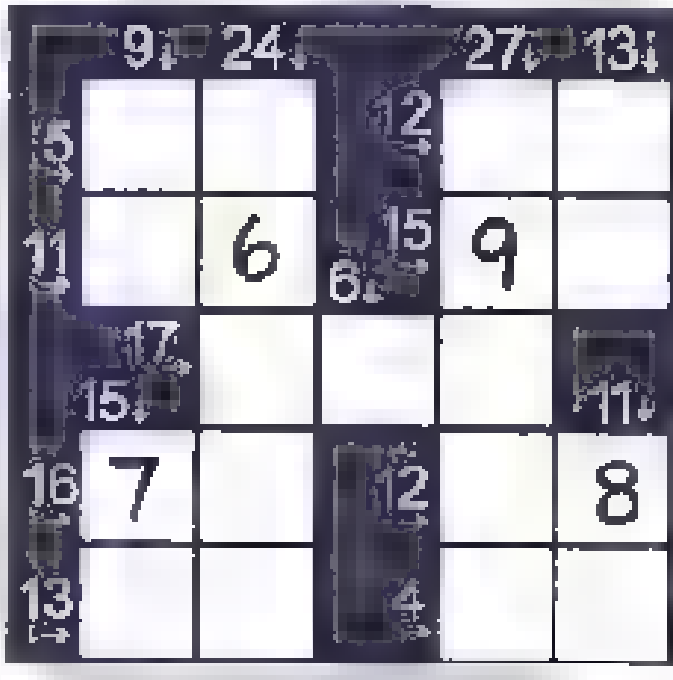
## Cross Sums

You must figure out what combination of numbers to use so that each column or row adds up to the totals shown in the white numbers. The white arrows show you in which direction you will be adding, and we have left you some numbers as hints. Better sharpen your pencil!

Here are a few simple rules:

- You are only adding the numbers in any set of white boxes that are touching each other.
- Use only the numbers 1 through 9. Each number can only be used once in each set.

**Remember, you need to think ahead a little bit. Each number has to be correct both across and down!**




## WORDS TO KNOW

***Fibonacci series:*** A series of numbers that begins with 1, 1, 2, 3, 5, 8, 13, and so on, where the two first numbers add up to the third (1+1=2), the second and third numbers add up to the fourth (1+2=3), and so on. This sequence was devised by Leonardo Pisano Fibonacci, a mathematician who lived in Pisa, Italy, between 1170 and 1250.

# It's My Favorite!

Solve each of the following problems. Watch the signs carefully! Shade each answer on the number grid to discover the hidden message. HINT: When there's an equation with parentheses ( ) around two numbers, do the part of the problem inside the parentheses first.

## HIDDEN MESSAGE:

**Professor Proof's bumper sticker tells about his favorite kind of calculation.**

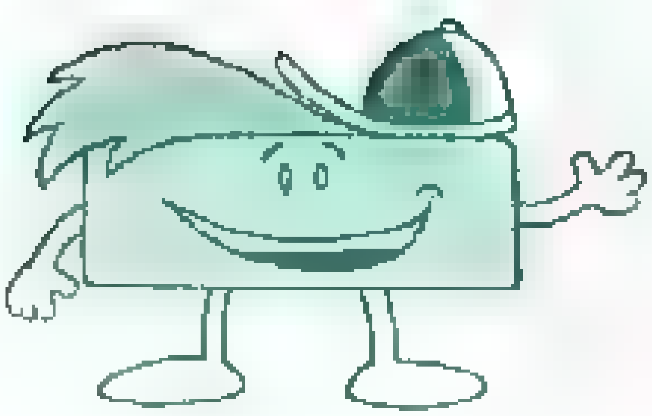
| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| [illegible] | 38 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 55 | 56 | [illegible] | 58 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | 66 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 73 | 74 | 75 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |

1 x 1 = ______
2 + 1 = ______
2 + 2 = ______
12 - 8 = ______
3 + 3 + 1 = ______
5 x 2 = ______
22 - 11 = ______
22 - 11 + 5 = ______
7 + 7 + 7 = ______
25 - 2 = ______
25 x 1 = ______
18 + 9 = ______

15 + 15 = ______
(15 x 2) + 4 = ______
(18 + 18) + 1 = ______
(14 x 3) - 8 = ______
(12 x 4) - 5 = ______
(12 x 4) - 1 = ______
25 x 2 = ______
52 - 1 = ______
(25 x 2) + 2 = ______
(17 x 3) + 2 = ______
(17 x 3) + 3 = ______
5 x 11 = ______

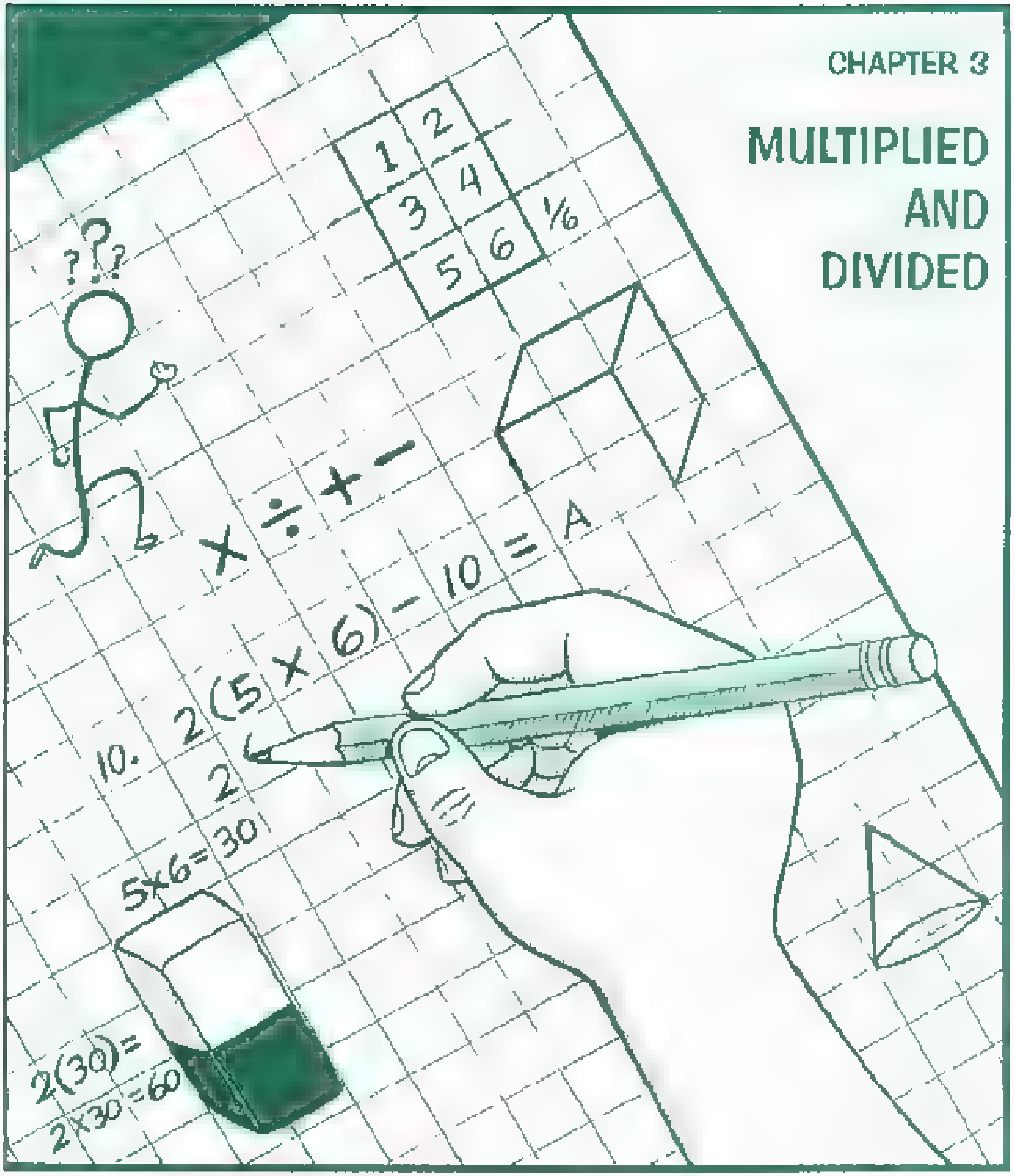
29 x 2 = ______
(60 x 2) - 60 = ______
16 + 16 + 16 + 16 = ______
80 - 10 = ______
(10 x 7) + 3 = ______
(10 x 7) + 7 = ______
(10 x 8) + 1 = ______
41 + 41 = ______
41 + 41 + 1 = ______
(84 x 2) - 84 = ______
(10 x 8) + 8 = ______

# NOTES



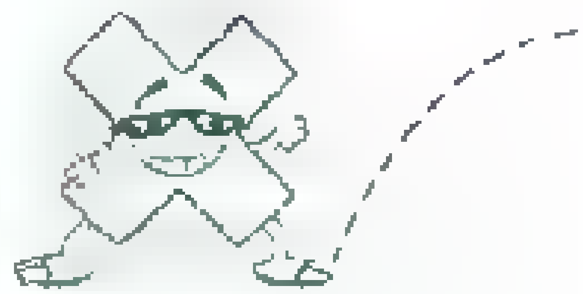
CHAPTER 3

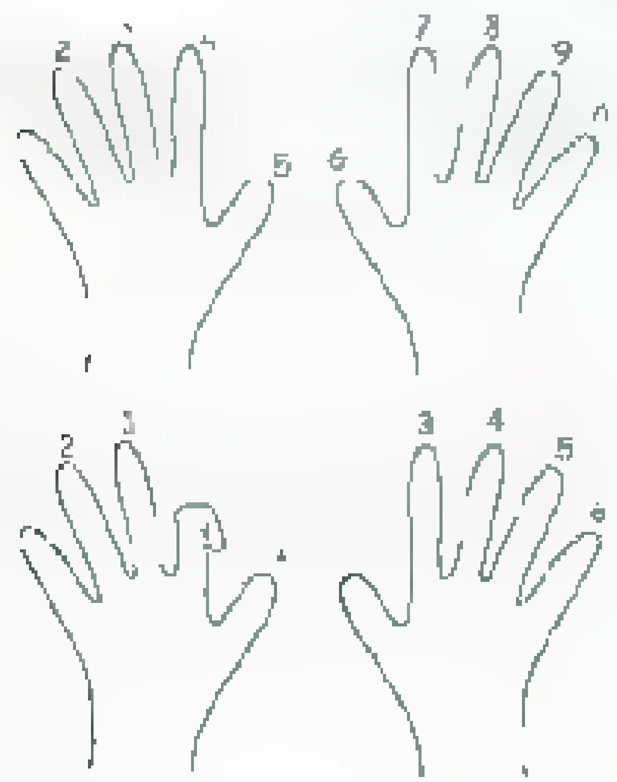
# MULTIPLIED AND DIVIDED



## MIGHTY MULTIPLICATION

M[illegible] shortcut to addition. Let's say [illegible] with apples, and each [illegible] seven apples. If [illegible] addition, you [illegible] problem, 7 [illegible] + [illegible] + [illegible] know how to [illegible] [illegible] same number [illegible] apples [illegible]





### Multiplying on Your Fingers

Here's a trick you can try with your friends—show them how to multiply by nine on your fingers. Hold your hands out in front of you. Then follow the following example.

To multiply 4 × 9, bend down the fourth finger from the left. The number of fingers to the left of the bent finger represent the "tens" digit and the number of fingers to the right represent the "ones" digit, so the answer is 36. This trick works up to 9 × 9 = 81.

**• To Multiply 9 by Any Digit •**

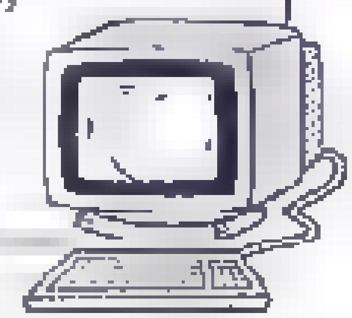
Can you figure out how the finger multiplication trick works? Here is another way to look at it—try to multiply 9 × 6:

The "tens" digit: 6 – 1 = 5
The "ones" digit. 9 – 5 = 4

Therefore, 9 × 6 = 54. Right?

**Play Five in a Row**

Improve your multiplication skills by playing Five in a Row, a game of multiplication tic-tac-toe, at http://nu[illegible] [illegible] applets/[illegible] 00023/welcome.html



## Russian Peasant Multiplication

In [illegible] interesting technique for multiplication—**halving, doubling, and adding numbers**. [illegible] bers that you need to multiply keep [illegible] one as you divide the other [illegible] ignore any remainders [illegible]. If the number in the halved column is odd (including the original value), mark the doubled number [illegible] [illegible] cored? It will make a lot more sense with an example. Let's take 22 × 44.

In the first column, there are three odd numbers: 11, 5, and 1. Their corresponding doubles are 88, 176, and 704. 88 + 176 + 704 = 968. So 22 × 44 [illegible]. If you don't believe this, check it by using [illegible].

| HALVE THIS COLUMN | DOUBLE THIS COLUMN | MARK THE DOUBLED NUMBERS WHERE THE HALVED NUMBER IS ODD |
|---|---|---|
| 22 | 44 | |
| 11 | 88 | + |
| 5 | 176 | + |
| 2 | 352 | |
| 1 | 704 | + |
| | Sum = 968 | |

How about trying one for yourself? Try using the Russian peasant method to multiply 25 by 34.

| HALVE THIS COLUMN | DOUBLE THIS COLUMN | MARK THE DOUBLED NUMBERS WHERE THE HALVED NUMBER IS ODD |
|---|---|---|
| 25 | 34 | |
| | | |
| | | |
| | | |
| | | |
| | | |

Sum =

### Practice ChisenBop

ChisenBop is an ancient method of doing basic math using your fingers. Kids who are good at ChisenBop can add and subtract as fast as a calculator! Check out *www.klingon.cs.iupui.edu/~aharris/chis/chis/html* for pictures and lessons.

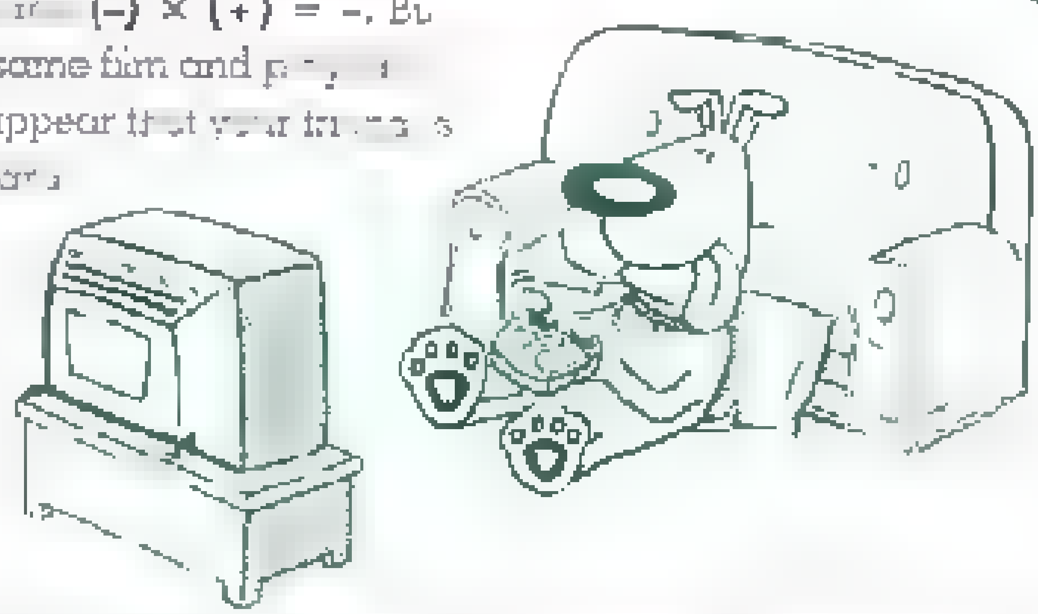
## Positive and Negative Numbers

In Chapter 2, you learned how to add and subtract positive and negative numbers. But what about multiplication? Do the same rules apply? Actually, they don't. In fact, the number line method doesn't work for multiplication. Instead, it might be helpful to imagine a video of a person who is walking backward and forward. People can walk backward and forward, and the video can be played forward or rewound.

**Walking forward is a positive action.**
**Walking backward is a negative action.**
**Film running forward is positive.**
**Film running backward is negative.**

Imagine that you videotape your friend walking forward. If you watch it on video played forward, you will see your [illegible] walking forward. This represents multiplying two positive numbers: (+) × (+) = +. Now, if you rewind the tape, [illegible] friend will seem to be [illegible] ward: (+) × ( ) [illegible].

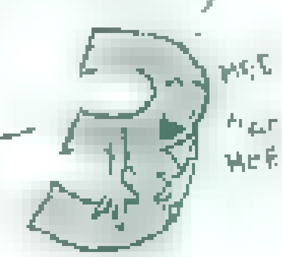
[illegible] imagine [illegible] your friend walking backward [illegible] the video forward. On [illegible] you will see your [illegible] walking backward: (–) × (+) = –. But what if you took that same film and played [illegible]? It would appear that your friend is [illegible] walking forward: ( ) × ( ) = +.



## FUN FACT

### To Make a Long Story Short

The nice thing is that these exact same rules work for division, so you don't need to learn any more new rules. In fact, these four rules are usually shortened to make them easier to remember. For multiplication and division, if the signs are the same, the answer is positive. If the signs are opposite, the answer is negative.



# Multiplication Boxes

Now you are ready for some [illegible] es. A multiplication box has [illegible] plying prob[illegible] [illegible] the first two numb[illegible]

The following multiplication box is filled out for you, with explanations on the side and bottom.

| | | | |
|---|---|---|---|
| -3 | 4 | -12 | -3 x 4 -12 |
| 2 | -6 | -12 | 2 x 6 12 |
| 6 | -24 | 144 | 6 x 24 144 |
| -3<br>x<br>2<br>=<br>-6 | 4<br>x<br>6<br>=<br>-24 | [illegible]<br>x<br>-12<br>=<br>144 | |

Now, complete each of the following multiplication boxes:

| | | |
|---|---|---|
| -4 | 5 | |
| -9 | | |
| | 30 | |

| | | |
|---|---|---|
| 7 | 10 | |
| | | -11 |
| | -10 | |

| | | |
|---|---|---|
| | | 21 |
| 5 | | 10 |
| | -6 | |

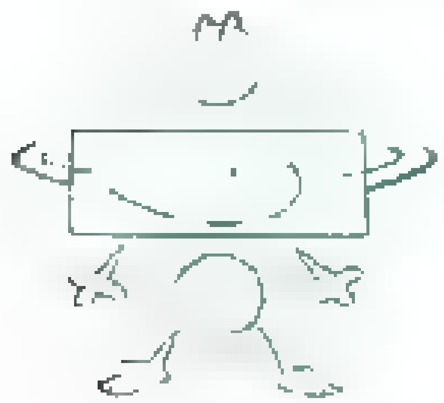
| | | |
|---|---|---|
| -3 | | -18 |
| | | |
| -21 | | 1008 |

## DAUNTING DIVISION

W[illegible] operation is the most [illegible] Problems like [illegible] to [illegible] you just memorize the [illegible] you [illegible] or take them [illegible] Problems like 542 ÷ 7 are harder. You have to learn how to solve them.

### WORDS [illegible] KNOW

**obelus:** [illegible] used to indicate division. The word comes from the Greek word obelos, for spit or spike, a pointed stick used for cooking. The symbol has been used as a division symbol since around 1650.



One of the things that makes division difficult is that unlike [illegible] and multiplication, even when you start with two whole [illegible] the answer is not always a **whole number;** it may be a **decimal** [illegible] adding or multiplying. When you add or multiply any two whole [illegible] answer is always another whole number [illegible] no exceptions. Mathematicians say [illegible] **closed** [illegible] and [illegible]

### Musical Math

Use division to complete the ten problems. Then cross off the answers in the box below. The remaining letters will spell out the answer to the following riddle:

**What comes before a tuba?**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Y 31 | A 45 | U 47 | O 16 | R 19 | N 12 | Y 21 | M 29 |
| R 44 | E 13 | B 15 | R [illegible]3 | M 25 | G 8 | A 22 | W 14 |

$5\overline{)95}$ $4\overline{)72}$ $3\overline{)87}$ $6\overline{)84}$

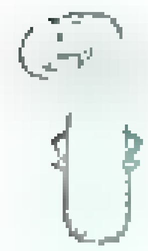
$7\overline{)175}$ $3\overline{)141}$

$8\overline{)168}$ $6\overline{)264}$

$8\overline{)264}$ $6\overline{)186}$

## Divisibility Rules

[illegible] number? Yes there is. There is a set of rules, called Divisibility Rules, that tell whether the answer of a division problem will be a whole number without actually having to do the long division.



**Dividing by 1:** If you divide any whole number by 1, you always get a whole number.

**Dividing by 2:** Even numbers "evenly" divide into 2. Odd numbers divide into 2 with an "odd one out."

**Dividing by 3:** Add up the digits (twice, if necessary): if the sum is divisible by 3, then the number is too. Let's say you need to divide 123: 1 + 2 + 3 = 6. 6 is divisible by 3, so 123 is divisible by 3. Another example: 678678. Add 6 + 7 + 8 + 6 + 7 + 8 = 42. 4 + 2 = 6, which is divisible by 3. That means 678678 is divisible by 3.

**Dividing by 4:** Look at the last two digits. If they are divisible by 4, the number is as well. For example, the last two digits of 2357924 are 24, which is divisible by 4. Therefore, 2357924 is divisible by 4.

**Dividing by 5:** If the last digit is a 5 or a 0, then the number is divisible by 5. For example, 2357925 is divisible by 5, because the last digit is a 5.

**Dividing by 6:** If the number is divisible by both 3 and 2, it is divisible by 6 as well. For example, 2157924 is divisible by 6 because it is even (divisible by 2) and the digits add up to 30, which is divisible by 3.

**Dividing by 7:** To find out if a number is divisible by 7, take the last digit, double it, and subtract it from the rest of the number without the last digit. If you get an answer divisible by 7 (including 0), then the original number is divisible by 7. If you don't know the new number's divisibility, you can apply the rule again. For example, 161 is divisible by 7 because 2 × 1 (the last digit) = 2 and 16 − 2 = 14, which is divisible by 7.

**Dividing by 8:** If the last three digits of a number are divisible by 8, then so is the whole number. How do you check the last three digits? If the first digit is even, and the last two digits are divisible by 8, the number is divisible by 8. If the first digit is odd, subtract 4 from the last two digits: the number will be divisible by 8 if the resulting last two digits are. For example:

- **2448:** Check the last three digits, 448. Here, 4 is even and 48 is divisible by 8, so 2448 is also divisible by 8.
- **192:** Here, 1 is odd, so you need to subtract 4 from the last two digits: 92 − 4 = 88. 88 is divisible by 8, so 192 is as well.

**Dividing by 9:** Add the digits. If they are divisible by 9, then the number is as well. For example, 52866 is divisible by 9 because 5 + 2 + 8 + 6 + 6 = 27, and 27 is divisible by 9.

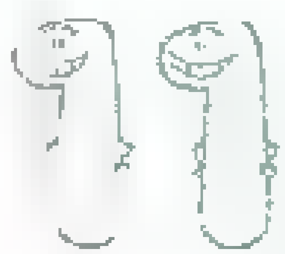
## WORDS 2 KNOW

***solidus:*** The slanted bar "/" used for fractions and division. During the Roman Empire, the solidus was a gold coin. On the reverse of the coin was a picture of a spear bearer, with the spear going from lower left to upper right. This spear became the symbol for fractions and division.

### A Problem with No Answer

There is one case in which division is not allowed. Do you know what it is? Try the following problem: 2 ÷ 0. Since division is the opposite of multiplication, this is the same as asking "What number times 0 will equal 2?" Any number multiplied by 0 is equal to 0, so it's impossible to have a number that, when multiplied by 0, will equal 2. That's why 2 ÷ 0 really does have no answer. Division by 0 is simply not allowed.

**Dividing by 10:** If the number ends in 0, it is divisible by 10.



**Dividing by 11:** Keep subtracting the last digit from the previous digits until you can tell if the resulting number is divisible by 11. For example, 645634 is divisible by 11 because 64563 − 4 = 64559, 6455 − 9 = 6446, 644 − 6 = 638, 63 − 8 = 55, and 55 is divisible by 11.

**Dividing by 12:** Check for divisibility by 3 and 4.

**Stepping Stones** • [illegible] stepping stones. Each [illegible] Use the [illegible] rules to find a stepping stone [illegible] number [illegible] your marker on the stone. [illegible] and your partner [illegible] different sets of markers—if you use pennies, your partner can use nickels or dimes.)

Take turns rolling the dice and placing a marker. The first person to connect a line of markers (including along a diagonal) from one bank of the river to the other wins.

| | | | | | | |
|---|---|---|---|---|---|---|
| 270 | 1386 | 350 | 396 | 308 | 720 | 1056 |
| 990 | 336 | 864 | 1000 | 770 | 432 | 990 |
| 495 | 1155 | 297 | 1188 | 512 | 220 | 243 |
| 210 | 162 | 1001 | 840 | 363 | 1320 | 144 |
| 315 | 576 | 504 | 693 | 360 | 660 | 1331 |
| 168 | 1260 | 280 | 440 | 729 | 1440 | 1500 |
| 792 | 594 | 441 | 252 | 462 | 924 | 972 |

## PRIME NUMBERS

L[illegible] see Prime [illegible] is the best [illegible] prime numbers must be the best [illegible] right? Well, maybe prime numbers are not the best, but they are [illegible].

[illegible] figure out the first several prime numbers in your head. But as the numbers get bigger, it gets harder to know if they are prime (quick, is 91 a prime number?). For thousands of years, people have been curious about [illegible] find prime numbers. **The Greek mathematician Eratosthenes invented one such way more than two thousand years ago.**

### WORDS 2 KNOW

***prime number:*** A number that is divisible by just two different numbers, 1 and itself. Seven is prime because it can only be divided by 1 and 7. Eight is not a prime number because it can be divided by 1 and 8. It is also divisible by 2 and 4. The word **prime** comes from *primus*, the Latin word for "first."

### Is 1 a Prime Number?

Most mathematicians do not consider 1 to be either a prime or a composite number. It is a special number in multiplication because you can multiply any number by 1 and the answer is the same. Because of this property, 1 is called an **identity for multiplication.**

# The Sieve of Eratosthenes

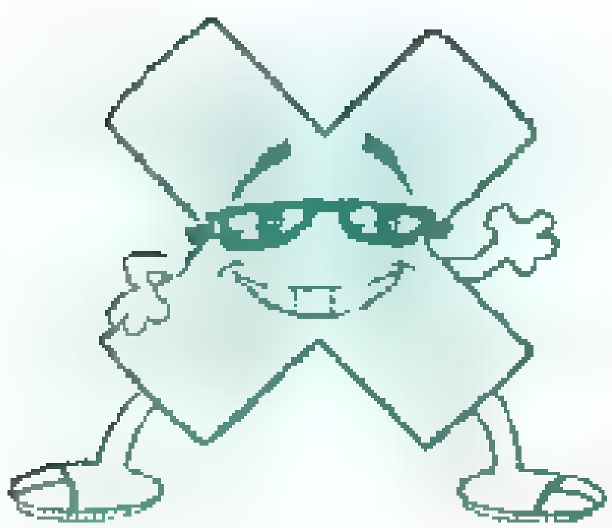
**The Sieve of Eratosthenes is a method for finding prime numbers on a number grid, and you can try it yourself by working with a number grid.** • First, circle the 2, then cross out all the numbers that are multiples of 2. • Then, circle the next number not crossed out (3) and cross out all numbers that are multiples of that number. • [illegible] circle the number closest to the beginning and cross out all its multiples until all the numbers are circled or crossed out. • When you are done, the circled numbers are **prime** numbers and the crossed out numbers are **composite** numbers.

### Who's That?

Eratosthenes [illegible] born [illegible] Cyrene, North Africa (now Libya) and died in Alexandria, Egypt. He was a well-rounded scholar who [illegible] many areas of math and science. He estimated the circumference of the Earth [illegible] 24,900 [illegible] miles [illegible] about [illegible]

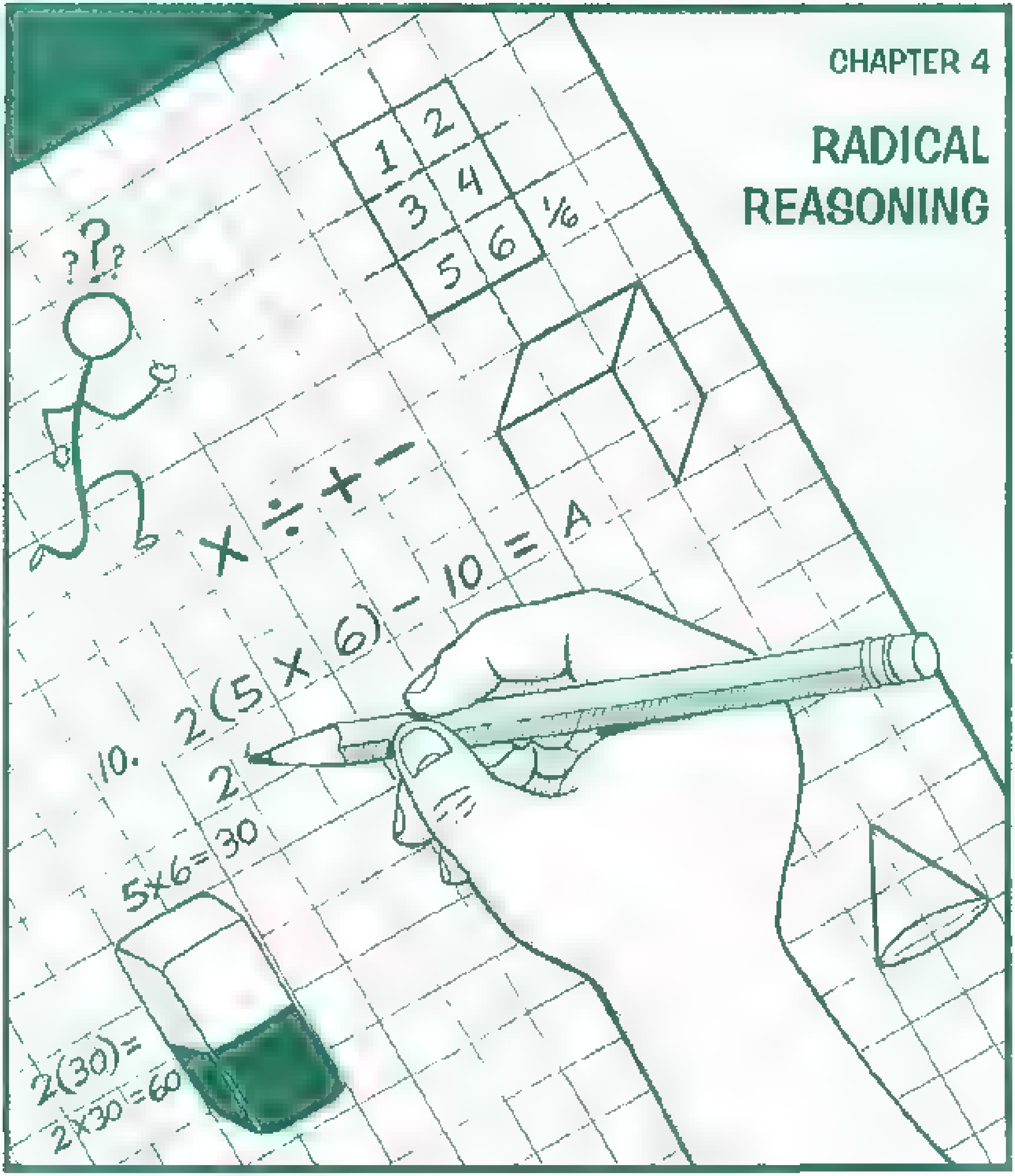
| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

# NOTES



CHAPTER 4

# RADICAL REASONING



## GROWING BY LEAPS AND BOUNDS

In the previous chapter, you learned that multiplication is simply repeated addition. But did you know that there is an operation that substitutes for repeated multiplication?

Here is an example: $3 \times 3 \times 3 \times 3 \times 3 = 3^5 = 243$. The **3** here is called the **base**, the **5** is the **exponent**, and the answer, **243**, is the **power**.

### A Calculator Tip

On many calculators, you use exponents by pressing a key marked $x^y$. Put in the base number first, 3, then press the $x^y$ key, then press the exponent [illegible] press the = key. You should see 243 as the answer. (A few calculators use the symbol ^ instead of the $x^y$ key.)

### The Allowance Scam

[illegible] may have [illegible] the famed allowance scam that [illegible] lucky math whiz kids [illegible] at their parents' expense. Here is how it works. Tell your parents that you are willing to accept two pennies for your allowance, but with one catch. Every day, the allowance should be raised to the next power. **If your parents agree, how much money will your allowance be by the end of the month (in thirty days)?**

| | |
|---|---|
| Day 1 | $2^1 = 2$ |
| Day 2 | $2^2 = 4$ |
| Day 3 | $2^3 = 8$ |
| Day 4 | $2^4 = 16$ |

If you were to be receiving your payments for thirty days, how much will you be entitled to receive on the last day of the month?

**$2^{30}$ pennies = 1,073,741,824 pennies or $10,737,418.24 —over ten million dollars!**

### It's Growing and Multiplying!

As you may have noticed with the allowance example, exponents on whole numbers [illegible] lead to some very large numbers. Many things in both nature and people's lives (though probably not your allowance) grow by repeated multiplication instead of repeated addition. Such growth is called **exponential growth**.

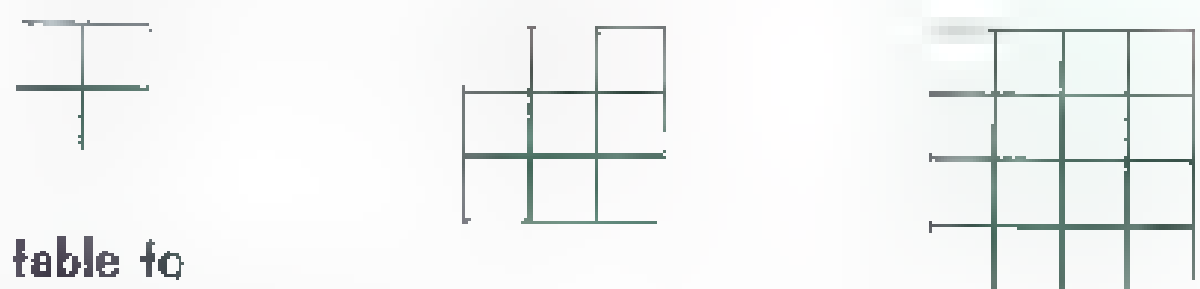
## SQUARES AND RADICALS

Square numbers are numbers that are the result of multiplying the same number twice. In other words, they can all be written with the exponent 2:

$1^2 = 1,\ 2^2 = 4,\ 3^2 = 9,\ 4^2 = 16,\ 5^2 = 25,\ 6^2 = 36, \ldots$

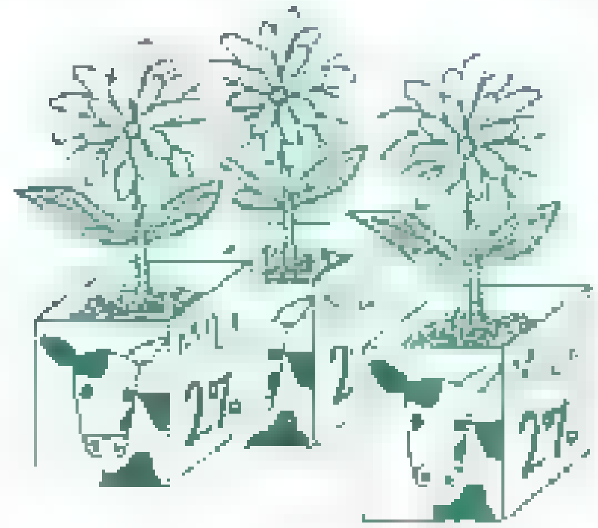
[illegible] can see what square [illegible] like whe[illegible] tiles [illegible] how [illegible]

$1 \times 1 = 1^2 = 1$ $\quad 2 \times 2 = 2^2 = 4$ $\quad 3 \times 3 = 3^2 = 9$ $\quad 4 \times 4 = 4^2 = 16$



**Complete the table to find more square numbers.**

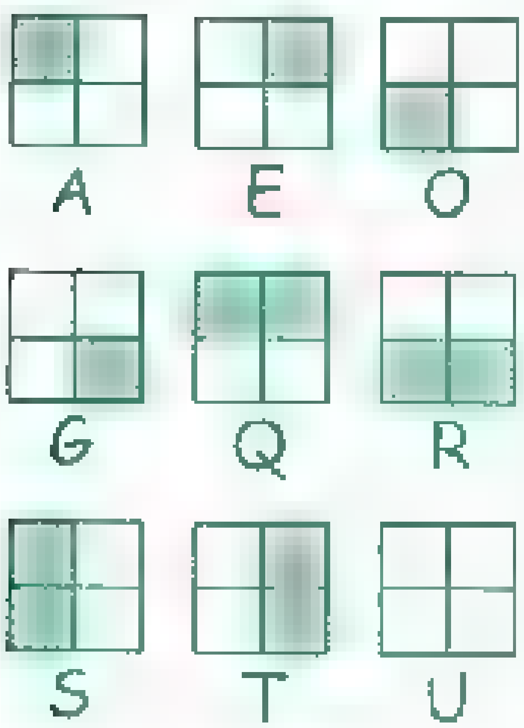
| | | | | | |
|---|---|---|---|---|---|
| $5^2$ | = | 25 | $13^2$ | = | |
| $6^2$ | = | | $14^2$ | = | |
| $7^2$ | = | | $15^2$ | = | |
| $8^2$ | = | | $16^2$ | = | |
| $9^2$ | = | | $17^2$ | = | |
| $10^2$ | = | | $18^2$ | = | |
| $11^2$ | = | | $19^2$ | = | |
| $12^2$ | = | | $20^2$ | = | |

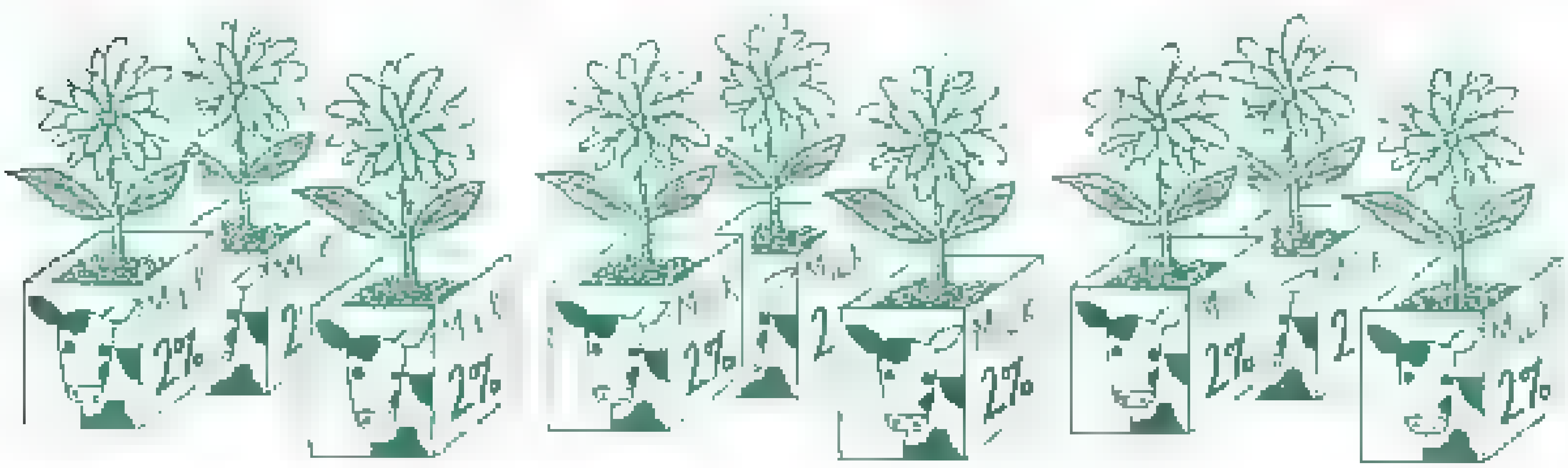


# Goofy Gardener

Use the decoder to figure out the answer to this riddle.

**Why did the mathematician plant his garden in milk cartons?**



## A Radical Sign

You know that subtraction is the opposite of addition and division is the opposite of multiplication. The operation [illegible] **squaring** also has an opposite, called the **square root.** A square root of a number is written using a **radical sign:** $\sqrt{36}$ means "What number must I square (multiply by itself) to get 36?" If you know your multiplication [illegible] could be able to find an answer fairly easily: it's 6, because $6 \times 6 = 36$. Now that you know [illegible] means, [illegible]

| | | | | | |
|---|---|---|---|---|---|
| $\sqrt{1}$ | = | 1 | $\sqrt{36}$ | = | 6 |
| $\sqrt{4}$ | = | 2 | $\sqrt{\text{[illegible]}}$ | = | 7 |
| $\sqrt{9}$ | = | | $\sqrt{64}$ | = | |
| $\sqrt{\text{[illegible]}}$ | = | 4 | $\sqrt{\text{[illegible]}}$ | = | 9 |
| $\sqrt{25}$ | = | | $\sqrt{100}$ | = | |

## The Square Root's Double

Did you know that $\sqrt{36}$ has another square root in addition to [illegible] What other two identical numbers [illegible] to 36? How about $-6 \times -6$? A positive [illegible] square roots that are [illegible]

### On Your Mark!

Answer each equation with a number between 1 and 10. Then read the phrases in order (read the number, too!) from least to most to find a popular way to count to the beginning of a race!

| | |
|---|---|
| $\boxed{\sqrt{9}}$ | to get ready, |
| $\boxed{-3+4}$ | for the money, |
| $\boxed{2^2}$ | to go! |
| $\boxed{4 \div 2}$ | for the show, |

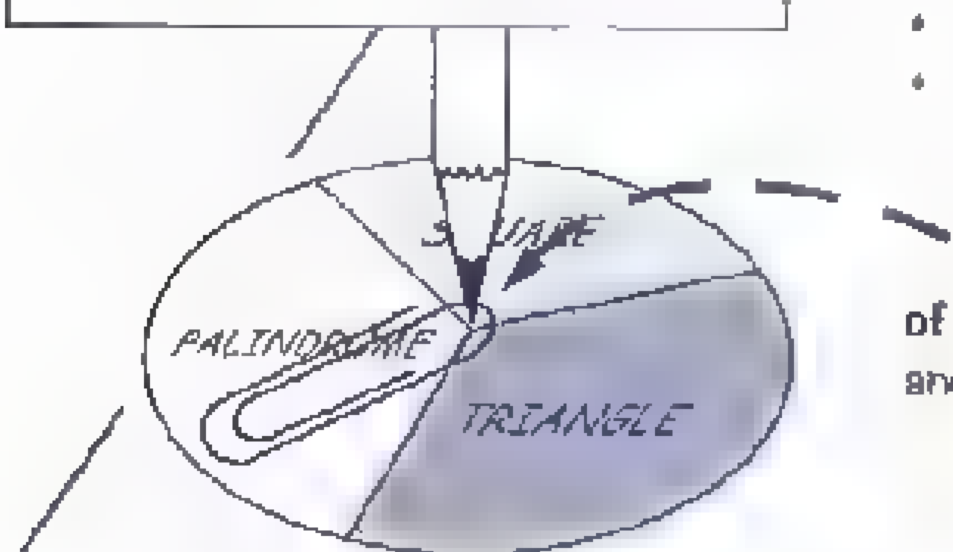
## FUN FACT

### Drawkcab (Backward) Numbers

Palindromes are words that read the same forward and backward. "Racecar" is a palindrome. Numbers can be palindromes if they are the same forward and backward, like 121. Here's how to make your own palindrome numbers:

- Start with any number: 49
- Reverse it: 94
- Add the two: 49 + 94 = 143
- Reverse the sum: 341
- Add again: 143 + 341 = 484

Presto! You've got a palindrome number: 484.




### THREE IN A ROW

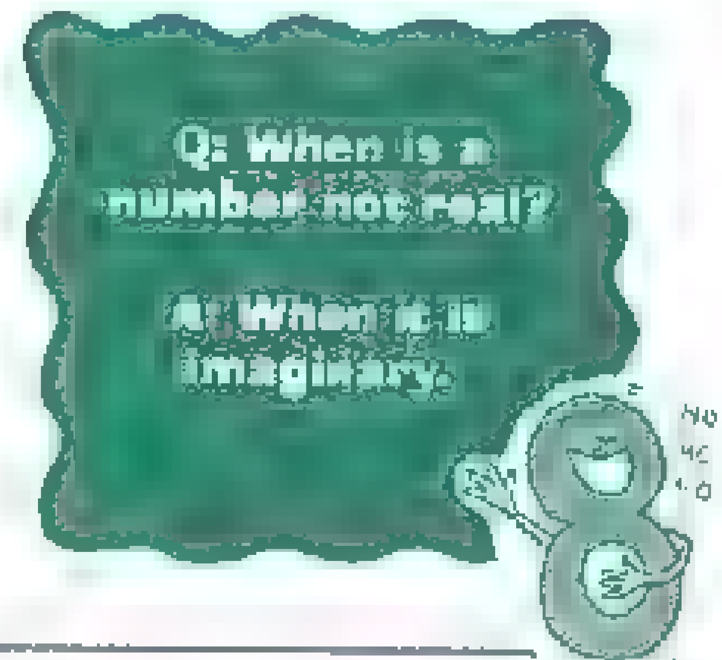
This is a game for two players. The goal of the game is to identify SQUARE, TRIANGULAR (divisible by 3), and PALINDROME numbers and be the first player to get three of your markers in a row. To play, you will need:

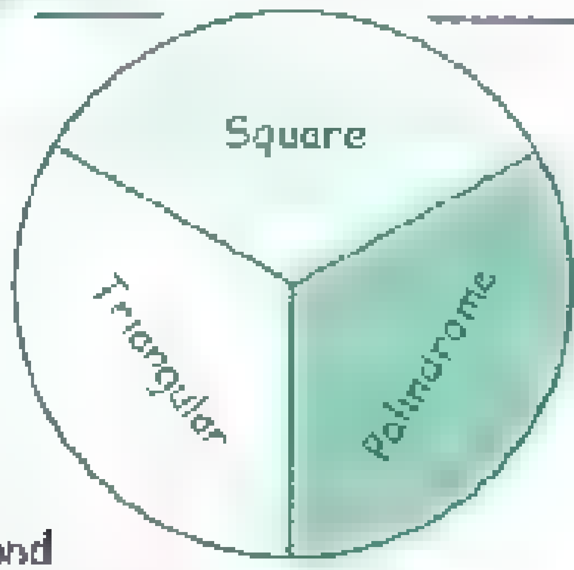
- A paper clip
- A pencil
- Two sets of different place markers (such as coins, buttons, or small candies)

Use the point of a pencil to hold one end of a paper clip at the center of the spinner and spin the paper clip.

### A Never-Ending Jar

An ancient Chinese porcelain jar has a sea, an island, two countries each with three mountains, and so on in *Anno's Mysterious Multiplying Jar* by Masaichiro and Mitsumasa Anno (Philomel Books, 1983). See how factorials can represent the total number of mountains and much larger numbers.




Player 1 spins the paper clip and places a marker on a number in the grid that matches the category picked (for instance, if the paper clip lands on "square," Player 1 can place a mark on 4, 9, 16, and so forth). Then, Player 2 does the same. Players take turns to see who can get three of their markers in a row first.

HINT: Some of the numbers fit more than one category.

| | | | | | |
|---|---|---|---|---|---|
| 1 | 105 | 99 | 49 | 21 | 88 |
| 3 | 16 | 11 | 22 | 25 | 55 |
| 100 | 111 | 91 | 77 | 9 | 15 |
| 4 | 101 | 78 | 66 | 6 | 44 |
| 64 | 10 | 242 | 141 | 121 | 33 |
| 81 | 45 | 28 | 131 | 36 | 144 |

# How Four Can You Go?

Four is a very powerful number. With exactly four fours (4 4 4 4), you can make lots of other numbers. Using addition, subtraction, multiplication, division, and square roots, as well as some parentheses (to show what to do first), see if you can come up with the numbers from 1 to 15. We've done 1 as an example, but feel free to do it again—most of these can be done more than one way.

1 = EXAMPLE: $4 \div 4 + 4 - 4 = 1$

2 =

3 =

4 =

5 =

6 =

7 =

8 =

9 =

10 =

11 =

12 =

13 =

14 =

15 =

**HINT:** Stumped? Don't forget that $\sqrt{4} = 2$!

# Your Number's Up

See how many common phrases or familiar objects you can think of that relate to the following numbers. Can you fill in all the blanks? We did a few to get you started.

1

2

3 LITTLE PIGS

4

5

6

7

8

9

10 FINGERS OR TOES

11

12

13

14

15

16

17

18

19

20

21

22

23

24 HOURS IN A DAY

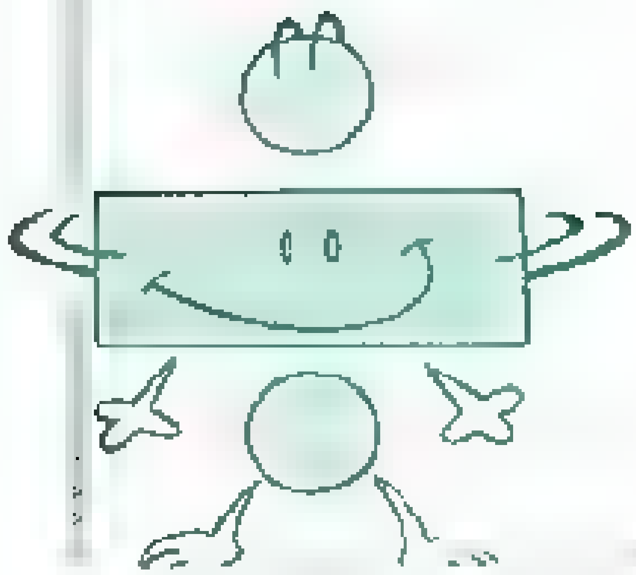
25

50

100

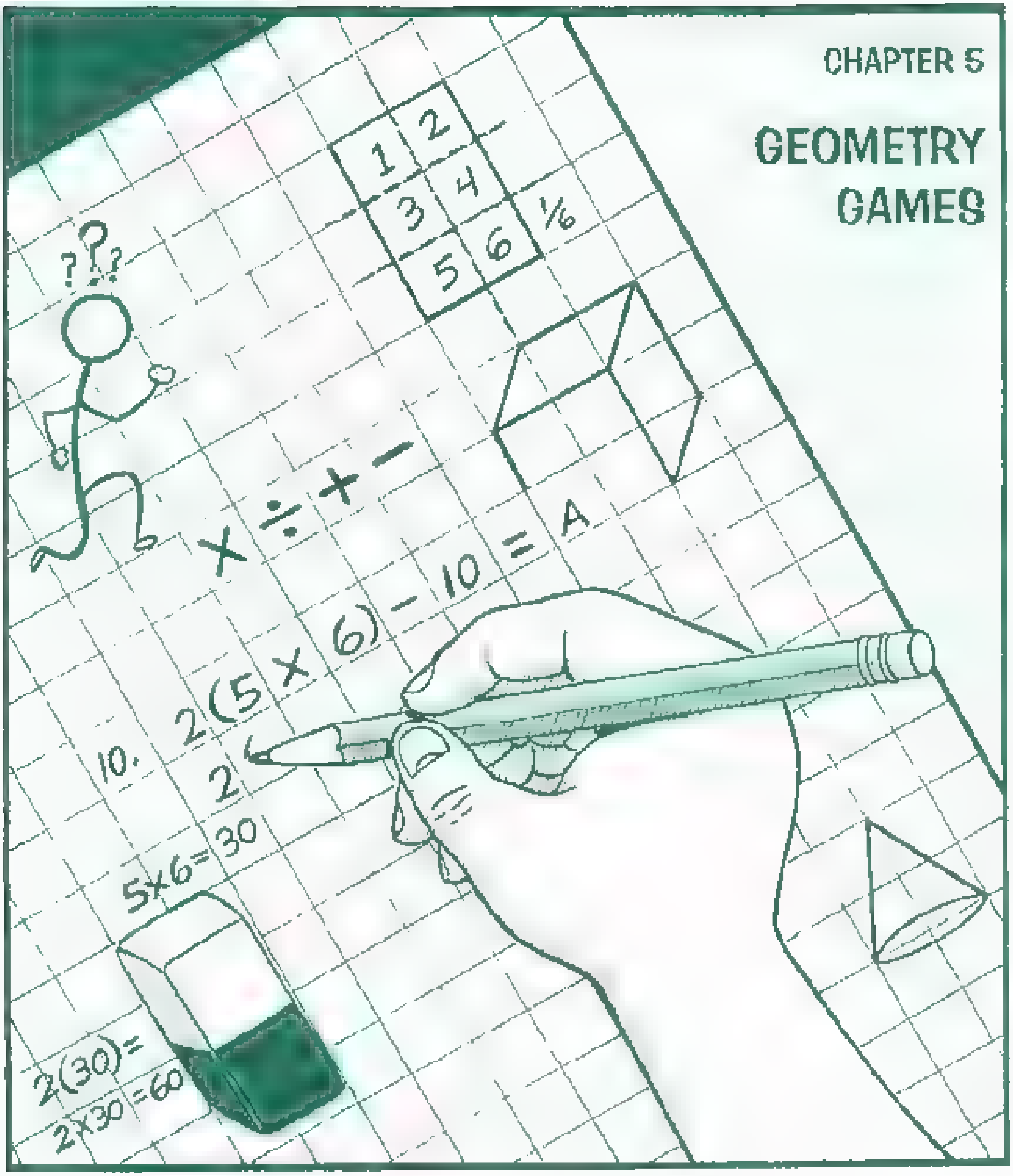
★ If you are stumped, you can check the answer key, but don't be surprised if your answers are different than ours!

# NOTES



CHAPTER 5

# GEOMETRY GAMES



## AN ANCIENT MATH

You may recall from Chapter 1 that numbers were invented [illegible] herdsman needed to keep track of how many sheep [illegible] We[illegible] herdsman may also have been [illegible] sheep had [illegible] branches of mathematics: geometry.

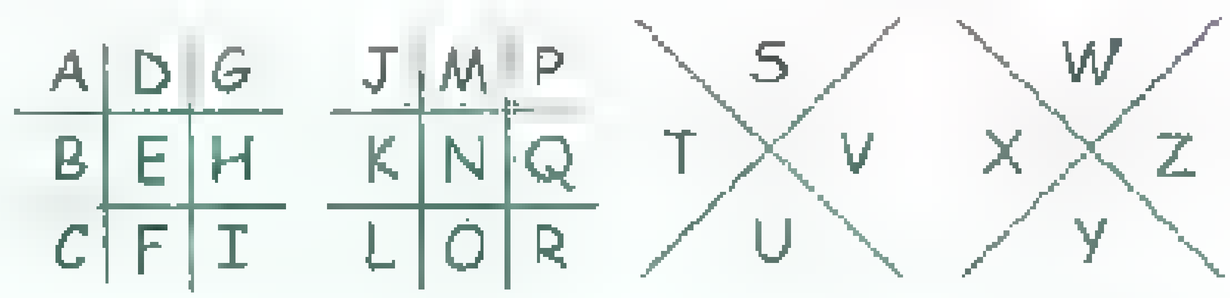
For [illegible] of years, many people from around the world have known the importance of geometry. The Chinese in the Far East, the Greeks in the Mediterranean, [illegible] Amer[illegible] geometry [illegible]

### WORDS 2 KNOW

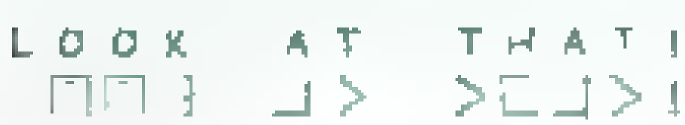
**geometry:** A study of physical shapes. Literally, **geometry** means "to measure the Earth."

### Shape Changers

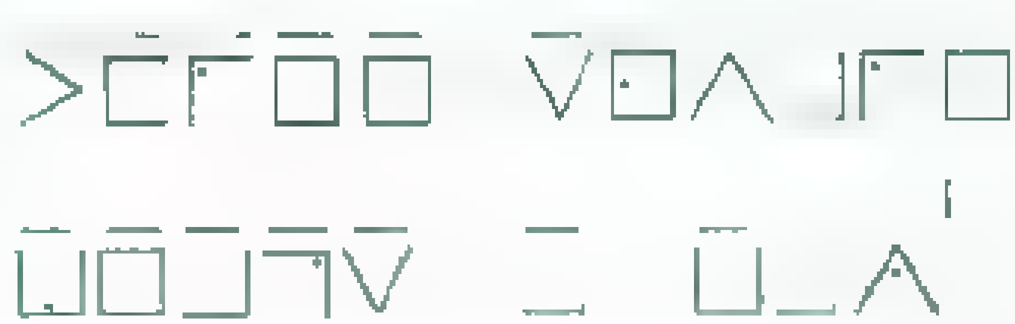
These are the shape-change codes for the puzzle you are about to solve.




Notice that each letter is found in a unique shape. To send a message, draw the outline of the shape each letter is in, including the dot, if there is one. For example, here is how to send the message LOOK AT THAT!:




Now use the decoder to figure out the answer to this riddle: WHAT DO GEOMETRY TEACHERS LIKE TO EAT?

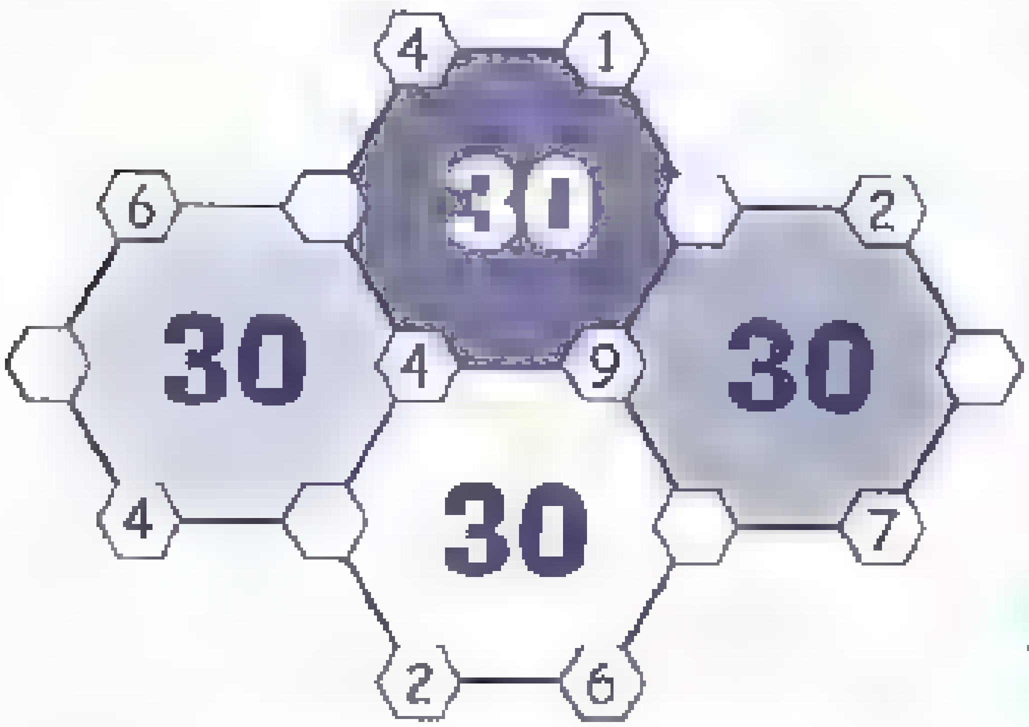


## BASIC GEOMETRY

U[illegible]derstanding ge[illegible] ng lots of words that are used to [illegible] metric figures and properties. Some of them [illegible] here. Although [illegible] begin your stu[illegible] g[illegible] know a lot of [illegible] between a long line and a short [illegible] between a square and a circle, and you [illegible] π (Chapter [illegible])

### Six-Sided Math

Here are four big hexagons that share some edges with each other. Use the numbers from 1 to 9 to fill in the empty spaces. When the puzzle is finished, the six numbers around every big hexagon should add up to 30.



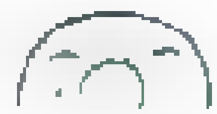

### WORDS 2 KNOW

***polygon:*** A geometrical figure with three or more sides. The term **polygon** is from the Greek roots *poli* (many) and *gonus* (knees). Greeks thought of a polygon as a shape of many angles, which look like bent knees.

**Q: Why did the cube cross from the second to the third dimension?**

**A: The second dimension was too square.**

# Hide in Plain Sight



**See if you can find this list of basic geometry terms hidden in the letter grid.**

**HINT:** The words in the grid will be "acting out" their meaning. For example, if you were searching for the word CIRCLE, expect to find the letters in a circular pattern, not a straight line! This means that some of the words will appear more than once in the puzzle, even if they only appear once in the list!

**EXTRA CREDIT:** Use bright-colored markers to run a line through each word you find. If you use a different color for each word, the patterns will be easier to see.

## Words to Search For

**Parallel**
Parallel lines are always the same distance apart. They never meet or cross over each other.

**Right Angle**
A 90° angle, like the corner of a book, or the capital letter L.

**Perpendicular**
Straight up and down, at right angles to the surface (a telephone pole is usually perpendicular to the road).

**Congruent**
Exactly equal in shape and area.

**Line**
The path made by a moving point, sometimes straight, sometimes curved.

**Rays**
A group of lines coming from a center.

**Bisect**
To part into equal halves.

**Rectangle**
A figure with four sides that has four right angles.

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| V | W | M | O | B | V | H | J | K | C | W | F | I | H | F |
| A | T | H | G | I | R | O | P | G | R | U | M | L | O | C |
| N | W | M | V | S | F | J | E | N | H | E | Z | E | H | M |
| G | J | V | / | / | / | V | R | O | H | N | C | N | F | O |
| L | F | K | Z | E | H | O | P | C | — | T | F | I | P | L |
| E | O | W | M | C | F | W | E | M | V | J | O | L | A | E |
| C | H | J | K | T | O | V | N | F | R | E | C | E | R | L |
| S | O | C | S | J | H | S | D | V | L | E | T | N | A | L |
| J | Y | K | Y | L | Y | K | I | O | G | N | A | I | L | A |
| M | J | A | A | A | Z | M | C | J | K | O | H | L | L | R |
| S | Y | A | R | A | Y | S | U | G | R | U | K | E | E | A |
| O | C | A | A | A | K | J | L | N | H | E | O | N | L | P |
| F | Y | J | Y | H | Y | K | A | O | H | N | F | I | O | C |
| S | O | F | S | O | J | S | R | C | — | T | V | L | M | O |
| V | P | E | R | P | E | N | D | I | C | U | L | A | R | J |

## Paper Folding

Origami is the art of folding paper. You can make lots of intricate figures with origami. [illegible] you may have seen an origami crane — one of the most popular origami animals.

The paper cranes are so well known because of Sadako, a Japanese girl who was dying of leukemia. Sadako heard an old legend: If a sick person folds one thousand origami cranes, the gods will grant her wish and make her healthy again. And so she set out on a quest to fold one thousand cranes. Unfortunately, Sadako died before completing her project. Ever since then, people have been making origami cranes in memory of this bright girl.

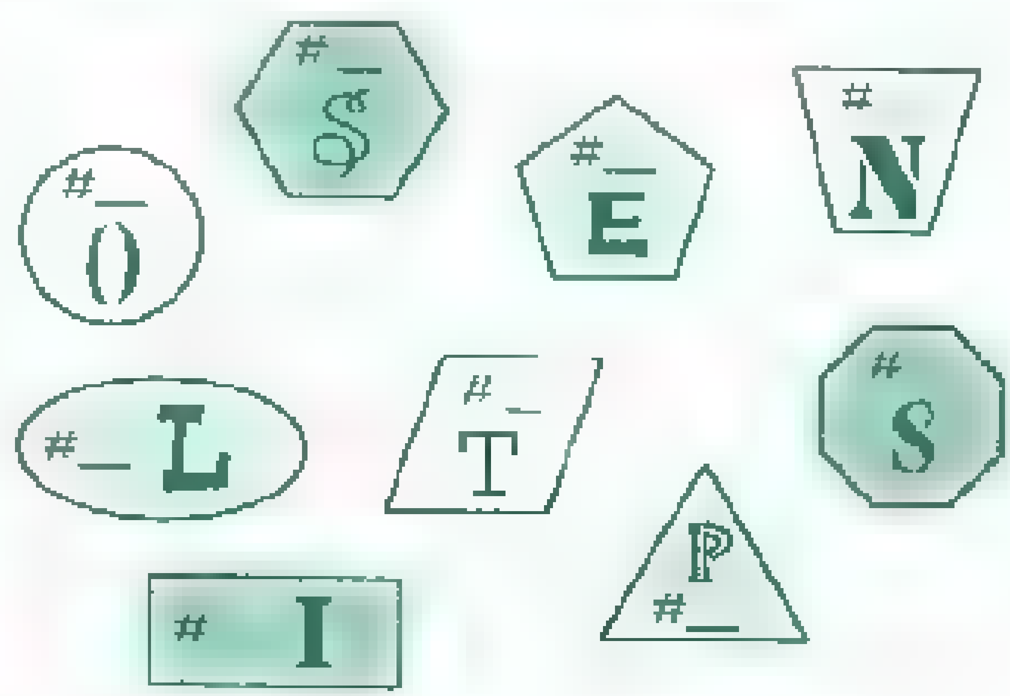
### Sadako and Her Paper Cranes

**Learn how to make paper cranes in memory of Sadako. Visit *www.personal.umich.edu/~adysart/origami/crane/* to learn how to make these paper creatures.**

### Life of the Party

[illegible]

1. Triangle
2. Circle
3. Rectangle
4. Trapezoid
5. Rhombus
6. Ellipse
7. Pentagon
8. Hexagon
9. Octagon

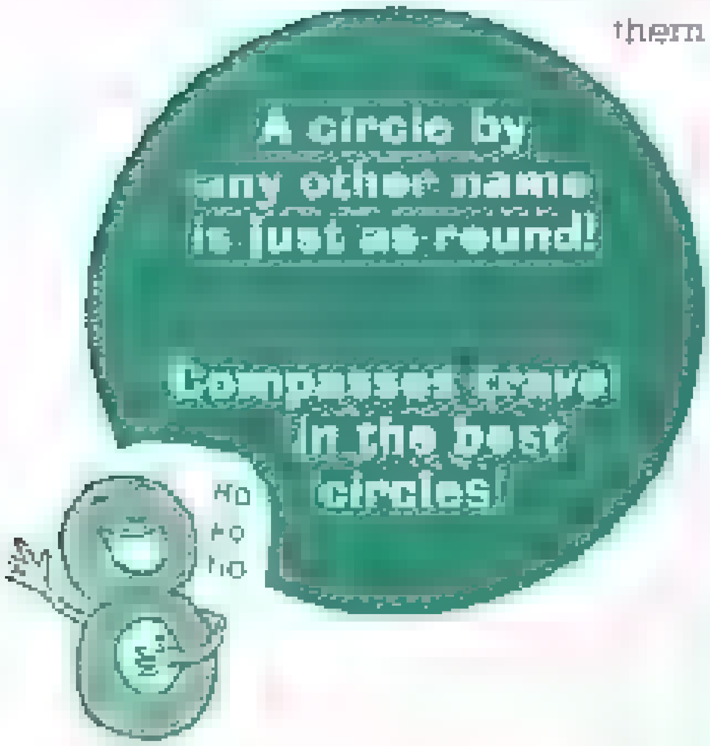



Without geometry, life would be:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |

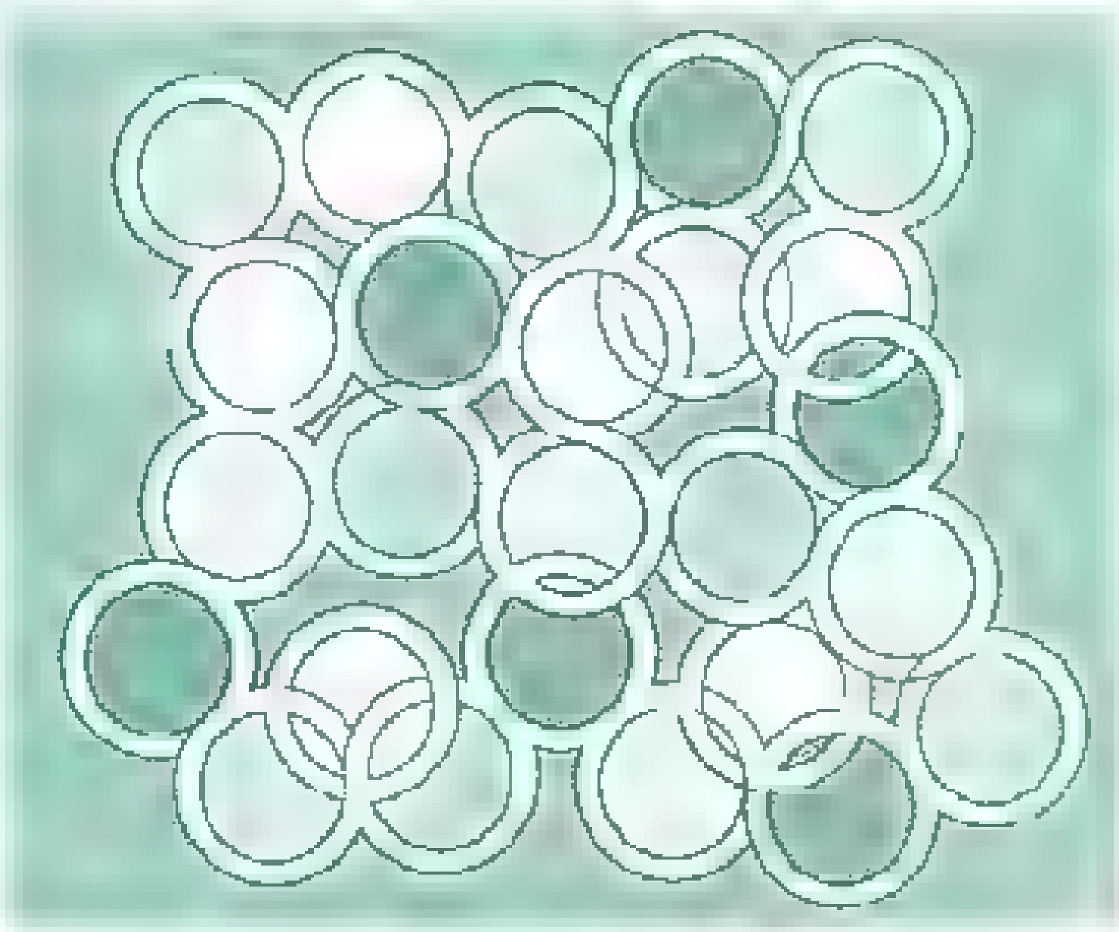
## CIRCULAR REASONING

You have al[illegible] studied [illegible] the basic [illegible]es in Chapter [illegible]. Circles are certainly mysterious. The center of the circle is exactly the same [illegible] to any point on the circle; that distance is [illegible] as the **radius.** Because circles cannot be measured by a ruler, you can't really divide them into [illegible]es or feet—[illegible] can divide them into degrees.




### Going Around in Circles

Find your way around the circles from start to end.



## FUN FACT

### 360 Circles

A circle is divided into 360 degrees. But did you know that a golf ball has 360 dimples? Coincidence? We think [illegible].

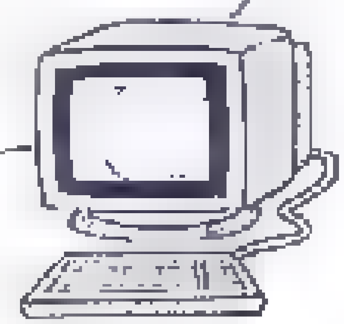
## The Möbius Strip

Who knew that a simple strip of paper would be the subject of a puzzle problem and that a man named Möbius would be forever remembered for it? Here is how a Möbius strip works.

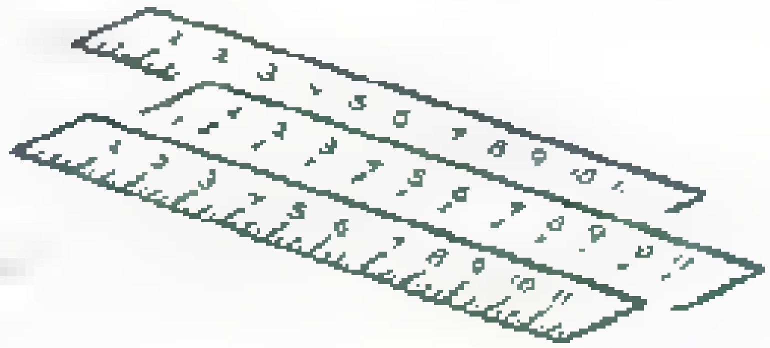
1. Take a long strip of paper—you can make one by cutting a piece of paper in several parts and taping them together to make one long strip.
2. Draw a car race on the top of the left end and one on top of the right end.
3. With one end in each hand, twist the paper once and bring the two ends together and tape them together with the ends overlapping. This special loop is called a **Möbius strip.**
4. Now, take a black crayon and draw a line down the middle of the paper strip; keep going until you get back to where you started. **Here is your first surprise.** Your pen mark went from the outside to the inside and back. If you drove a miniature car over your strip, you would race over the "top" and "bottom" and find the start line and the finish line are the same place.
5. Now, take a pair of scissors and cut along the line you drew. You might think you would get two separate rings, just as you would when you cut a normal ring of paper. **Here is your second surprise:** You get one really big loop with several twists in it. For the **third surprise,** cut this long loop in half again.

### It Is Mind-Boggling!

For an out-of-this-world experience, play tic-tac-toe on the surface of a torus or a Klein bottle instead of a flat piece of paper at *www.northnet.org/weeks*. The maze is also a lot of fun, but the chess game is for expert players only.

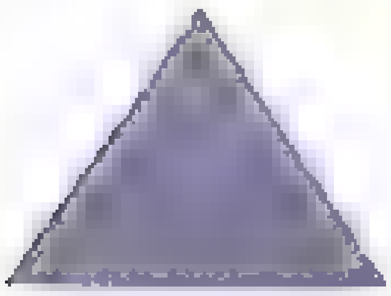


## TAMPERING WITH TRIANGLES



W[illegible]hat could be [illegible] des[illegible] length. But actually, there are [illegible] may vary by the **length of their sides**

**Triangles with three sides of equal length are equilateral triangles.**

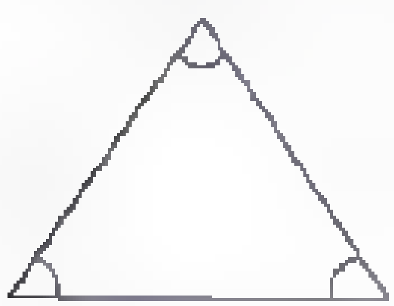


**Triangles with two sides of equal length are isosceles triangles.**

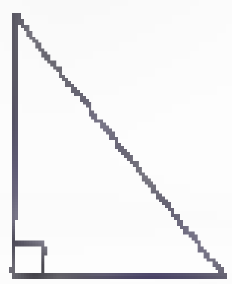
**Triangles with three different sides are scalene triangles.**

But that's not all. Triangles can also be sorted by their **angles:**

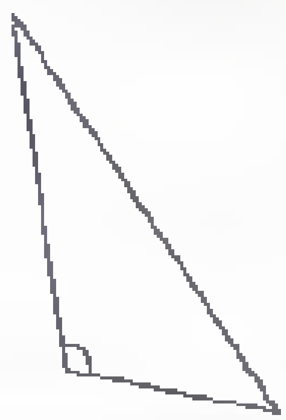
**A triangle is equiangular (and, therefore, equilateral) if all its angles are congruent (same).**



**A triangle is a right triangle if one of its angles measures 90 degrees (and two of its sides are perpendicular to each other).**



**A triangle is obtuse if one of its angles is larger than 90 degrees.**



**A triangle is acute if all three of its angles are smaller than 90 degrees.**

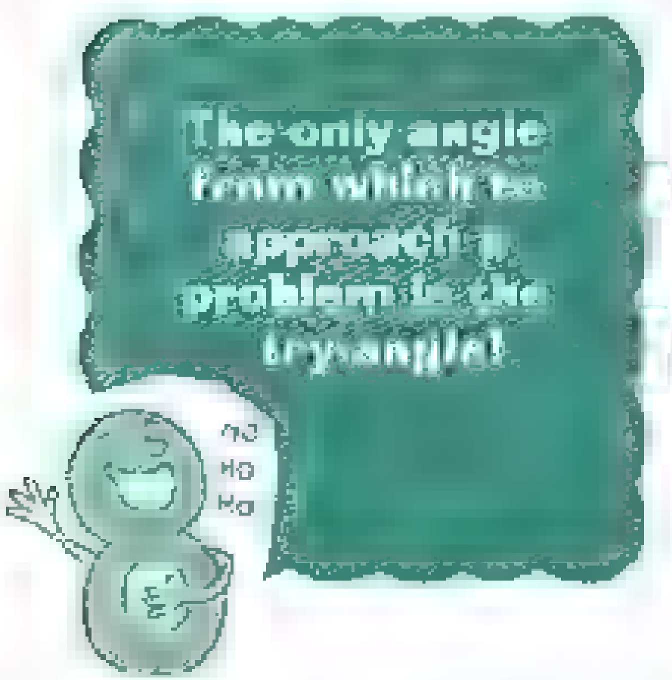
## Triangle Numbers

Triangular numbers are found in the number of dots that can be used to make an equilateral triangle (triangle with three equal sides). The following illustrations represent 3, 6, 10, and 15.

**Can you see a pattern here?**

1 + 2 = 3 1 + 2 + 3 = 6 1 + 2 + 3 + 4 = 10 1 + 2 + 3 + 4 + 5 = 15

Draw a picture or add numbers to find the next three triangular numbers




## The Last Straw

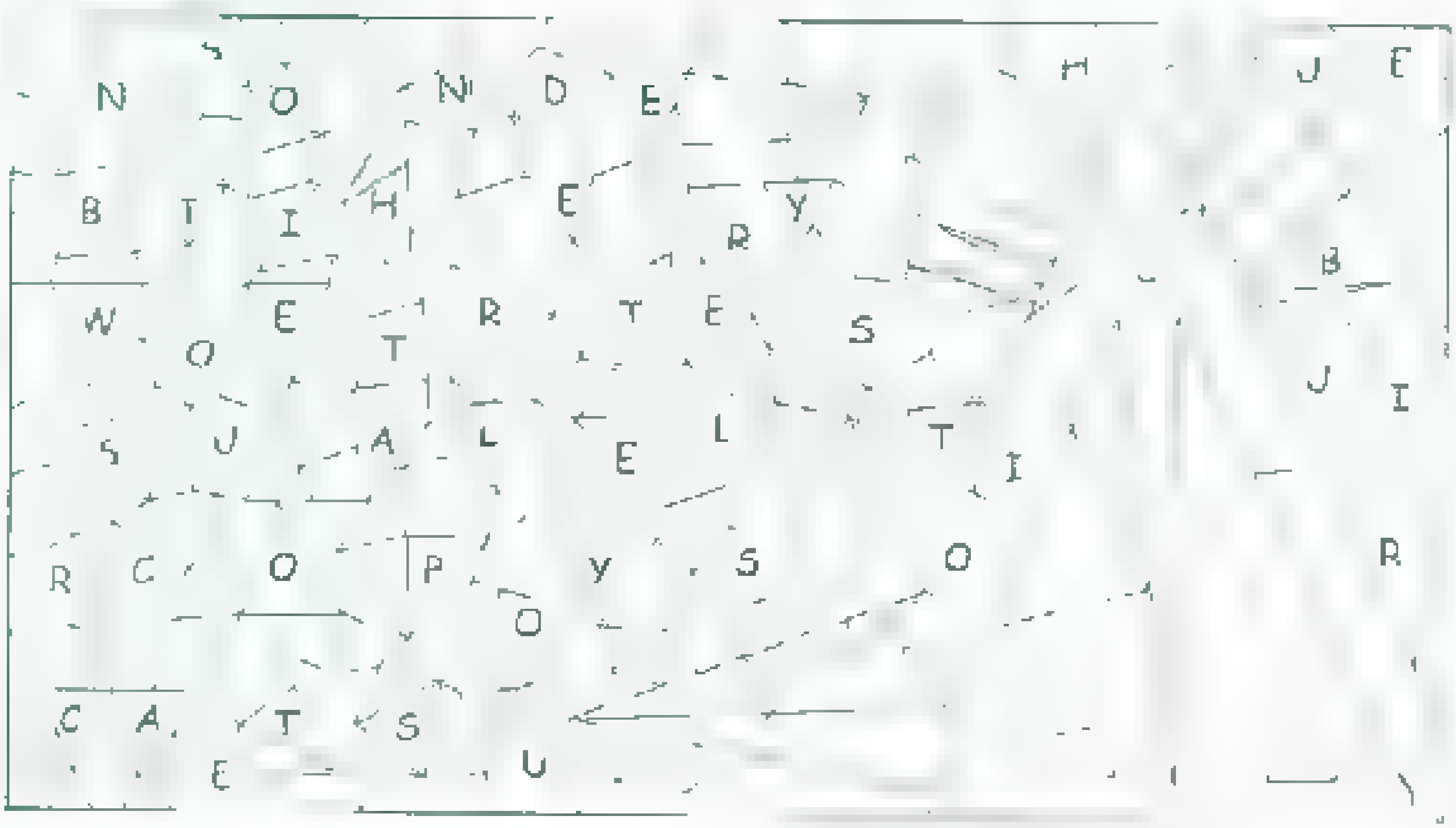
If you take three drinking straws, you can easily arrange them into one triangle that has three equal sides. Now, suppose you have six drinking straws. Can you arrange them into FOUR triangles, all with equal sides?

HINT: You may need some tape

# Get to the Point!

Color every triangle in this puzzle to find the answer to the following riddle:

If you have ten cats in a box and one jumps out, how many are left?

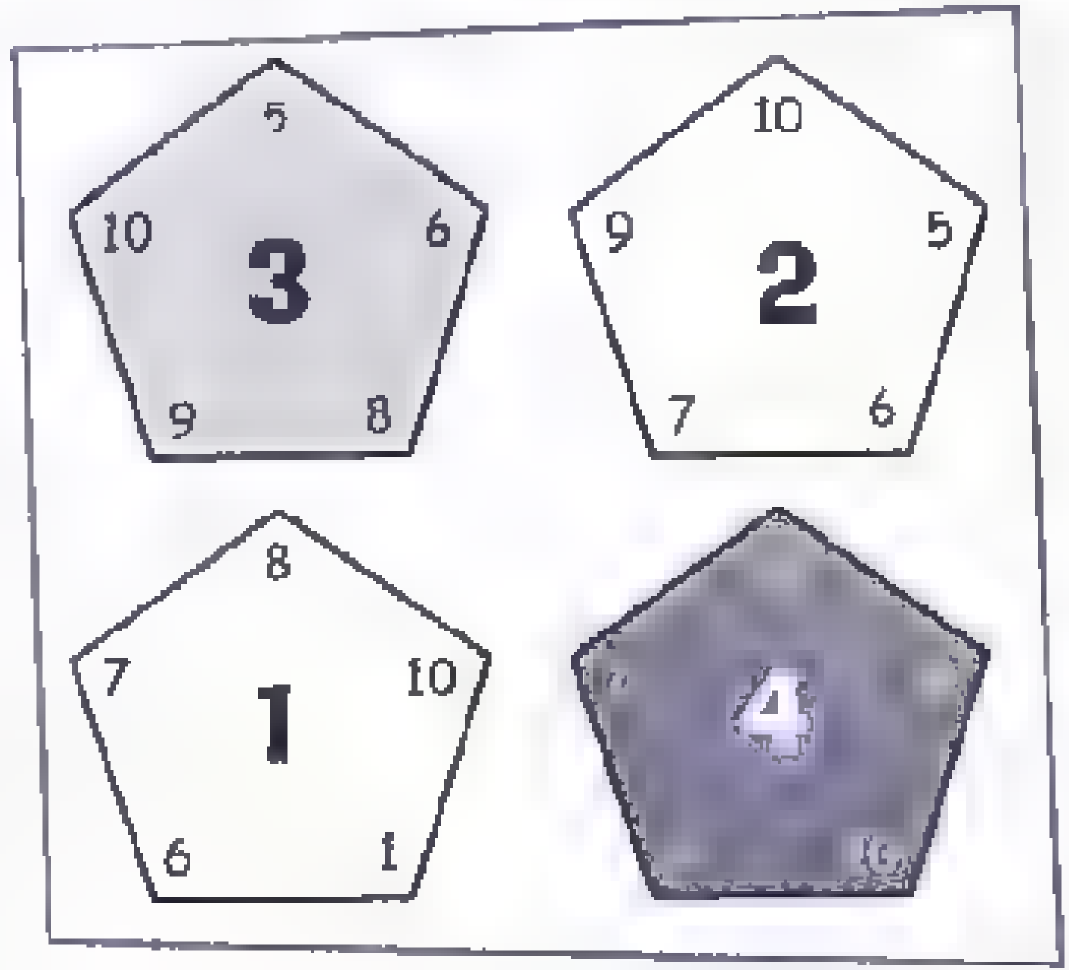


## Magic Pentagrams

**This puzzle is just for fun—there's no right or wrong answer!**

Choose a number found on one of the points of a pentagram. Now point to all the pentagrams on which this number can be found. Add the large number found in the center of those pentagrams. The sum will be the number that you chose!

Share the magic pentagram trick with a friend, or try making your own pentagram puzzle using different numbers.




### FUN FACT

### Gargantuan Geometry

Did you know that one of the largest pentagons in the world is right here in the United States? That's right—the Pentagon building just outside Washington, D.C., which serves as the headquarters of the Department of Defense, is so big that the National Capitol could fit into any one of its five wedge-shaped sections. To get around inside, there are 17.5 miles of corridors. Want to know more fascinating facts about this prominent polygon? Check out *www.fortamerica.com/about-the-pentagon.html*.

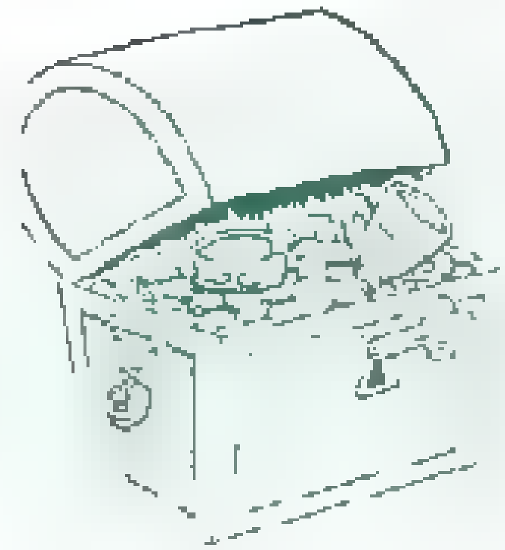
### SpiroGraphs

Create your own SpiroGraph design at *www.math.ucsd.edu/~dlittle/java/SpiroGraph.html*. There are suggestions for the different settings; try them all. Make the String of Pearls for someone special.

## ON A TREASURE HUNT

T[illegible] treasure [illegible] near the town of [illegible] Nevada. After [illegible] his granddaughter Penny went looking for the treasure.

She [illegible] very [illegible]. After [illegible] she decided [illegible] were ten possible [illegible] where he could have [illegible] money.



1. Under the dirt floor of his cabin.
2. In the hayloft of his barn.
3. In the Steele family cemetery.
4. High up in the branches of the Lone Pine Tree.
5. On top of Windy Hill.
6. In the cave to the east of Windy Hill.
7. In the old mine shaft.
8. At the top of the windmill.
9. In a pile of dinosaur bones.
10. Down at the bottom of the well.

Here is what else Penny knew:

1. Her father was afraid of ghosts and wouldn't want his money close to the cemetery.
2. Cash Steele did not like small dark places either.
3. If he buried the money, it would be to the north of his cabin.
4. If he hid it aboveground, it would be south of the cabin.
5. If he didn't hide it in the well, then he didn't hide it in the windmill, either.

That leaves one possible hiding place. [illegible]

*See the answer on page 129 for an explanation.*


mine shaft
barn
cemetery
Lone Pine
well
dino bones
cabin
cave
windmill
N
W
E
S
Windy Hill

## A Square Deal

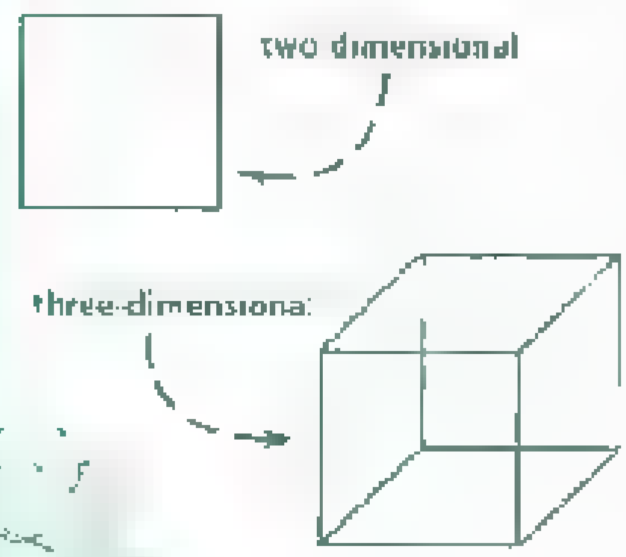
Begin at the whit[illegible] the dark box. Move [illegible] are [illegible] and reach all [illegible] only one time e[illegible]

**HINT: the pattern forms the outline of a letter of the alphabet.**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 2 | 4 | 3 | 1 | 3 | 4 | [illegible] | [illegible] |
| [illegible] | 1 | 2 | 4 | 3 | 4 | 2 | 1 | [illegible] |
| 1 | [illegible] | [illegible] | [illegible] | 2 | 3 | [illegible] | [illegible] | 3 |
| 3 | 4 | 3 | 2 | 1 | 2 | 3 | 1 | [illegible] |
| 4 | 3 | 2 | 1 | **4** | 1 | [illegible] | [illegible] | [illegible] |
| 3 | 4 | 3 | 2 | 1 | 2 | [illegible] | [illegible] | [illegible] |
| 1 | 2 | 4 | 3 | 2 | 3 | 4 | 2 | 3 |
| 2 | 1 | 2 | 4 | 1 | 4 | 2 | 1 | [illegible] |
| 4 | 2 | 4 | 1 | 3 | 1 | 4 | 2 | [illegible] |

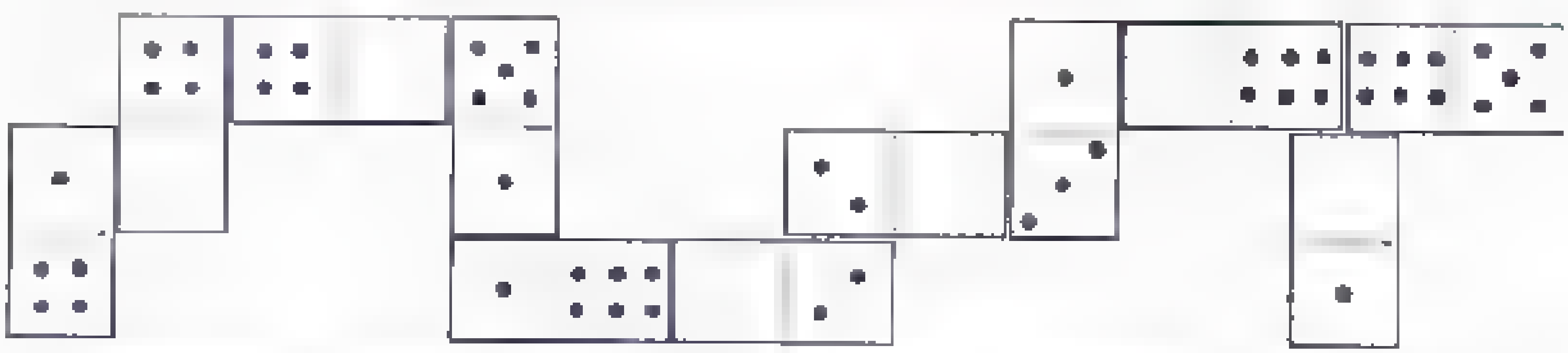
### Depth of the Matter

What's the difference between a square and a cube? The number of dimensions. Squares are **two-dimensional**: they have length and width. Cubes are **three-dimensional**: they have length, width, and depth.



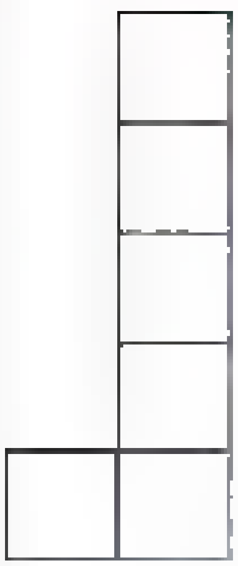

## The Domino Effect

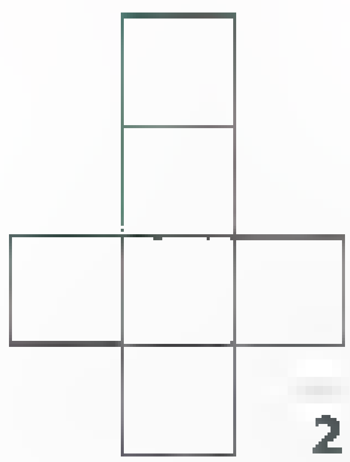
Do you know how to play d[illegible]? See [illegible] you can figure it out by [illegible] the missing squares. HINT: Each number from [illegible] can only be used once.

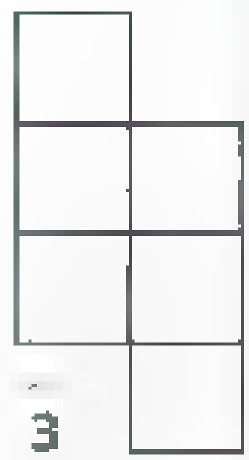


# Picture This

Look at the patterns below. Try to picture which shapes can be folded into a square box that has four sides, a top, and a bottom. Circle the correct answers. HINT: Having trouble picturing this in your mind? Use a ruler and a pen to draw the shapes onto heavy paper, cut them out, and see which shapes will actually fold into a box.


1


2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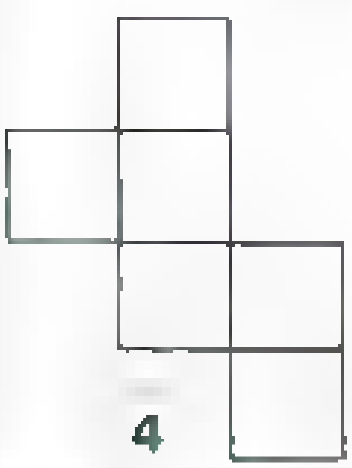
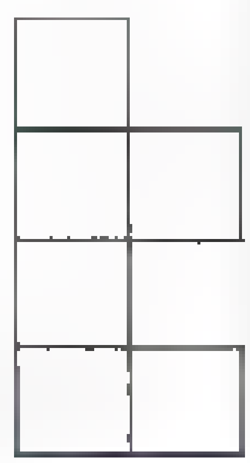

4


5


6

**Box 1**
Yes ☐
No ☐

**Box 2**
Yes ☐
No ☐

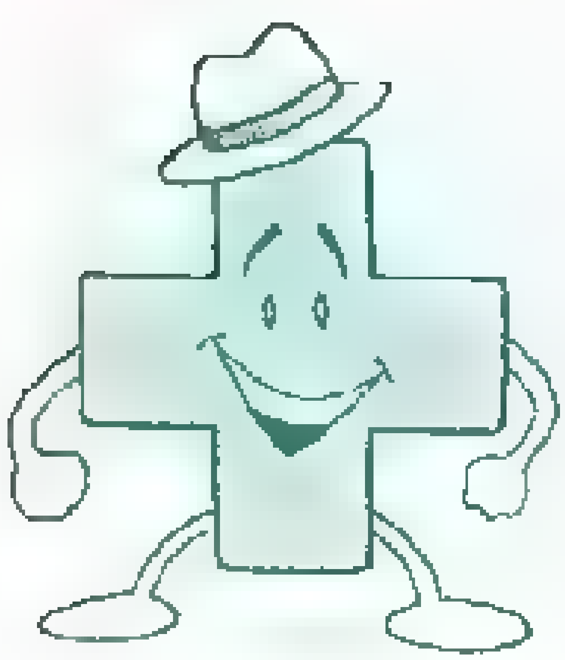
**Box 3**
Yes ☐
No ☐

**Box 4**
Yes ☐
No ☐

**Box 5**
Yes ☐
No ☐

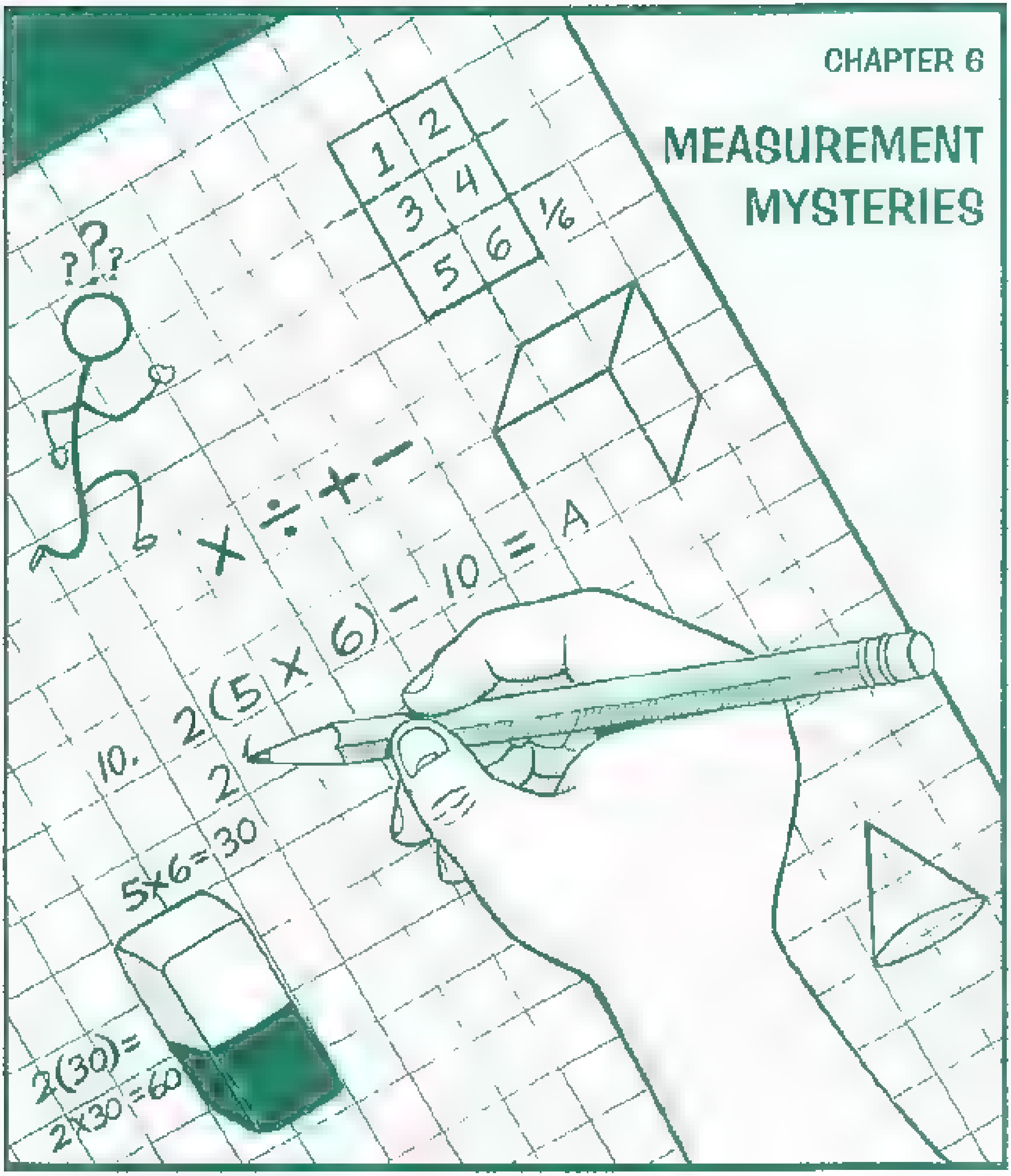
**Box 6**
Yes ☐
No ☐

# NOTES



# CHAPTER 6

# MEASUREMENT MYSTERIES



## MEASURING UNITS

Measurement is a very important part of everyday life. We measure all sorts of things. Here are just a few of them:

- **Size and distance:** How tall you are; how far away your school is
- **Weight:** How much do you weigh; how heavy is your backpack
- **Area:** How large is your apartment
- **Volume:** How much water is in your bathtub
- **Time:** How long does it take you to read this paragraph; how old you are

**FUN FACT**

***The Guinness Book of World Records***

Would you like to know the height of the tallest man who ever lived, the age of the oldest person on Earth, or the longest time spent riding a roller coaster? The answers to these questions—and *many* more—can be found in *The Guinness Book of World Records*. Check out the silliest records ever set, like the person who caught the most paintballs in two minutes and the person who kissed the most cobras (eleven of them!).



### Two Different Systems

There are two main systems of measurement in the world today. In America, we use the **English system**, which involves units such as inches, feet, yards, and miles for measuring lengths; ounces, pounds, and tons for measuring weights, and cups, pints, quarts, and gallons for measuring liquid volumes.

Most of the rest of the world (including, more and more England) uses the **metric system.** In this system, lengths are measured in millimeters, centimeters, meters, and kilometers, weights in grams and kilograms; and liquid volumes in milliliters and liters. The metric system is based on the base of 10.

## A Metric Mnemonic

A **mnemonic** is a memory trick to help you remember tricky information. The metric system is based on factors of 10, but the words used to represent each multiplication (or division) by 10 are easy to confuse. Here is a mnemonic device to remember the order of the prefixes in the metric system.

**Kilo ↔ Hecto ↔ Deka ↔ unit ↔ Deci ↔ Centi ↔ Milli**

Each arrow represents a factor of 10. When you move to the right, the measure gets smaller (divide by 10) for each arrow. When you move to the left, the measure gets larger (multiply by 10).

### Words to Know

***metric prefixes:*** A prefix is a part of the word that is added on at the beginning of the root. The following prefixes are added to "meter," "gram," and "liter" to form metric measurements.

**Milli**– 1/1,000
**Centi**– 1/100
**Deci**– 1/10
**Deka**– 10
**Hecto**– 100
**Kilo**– 1,000

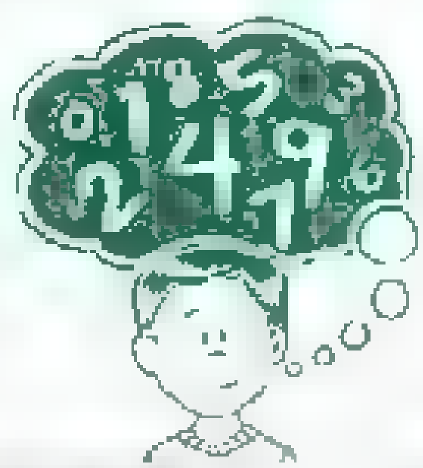
### Here's a story to help you remember the order of the names of the prefixes:

During medieval times, Kings considered themselves to be above all other Humans, and Humans considered themselves to be above the Dragons. These are the mighty and all are above the basic unit. Below the basic unit are Dogs, which considered themselves to be above Cats, which considered themselves to be above Mice.

### "Three Blind Mice" in Metrics

You can sing the metric hierarchy to the beginning of the tune of "Three Blind Mice":
Kings, Humans, Dragons,
Dogs, Cats, Mice

## Convenient or *Confusing?*



Many people believe that the English system is needlessly confusing because it's hard to convert from one measurement to another, and that we should switch to the metric system. But others believe that because we have grown up with the English system, we should continue using it because it's easier than learning a new one. **Which group do you agree with?**

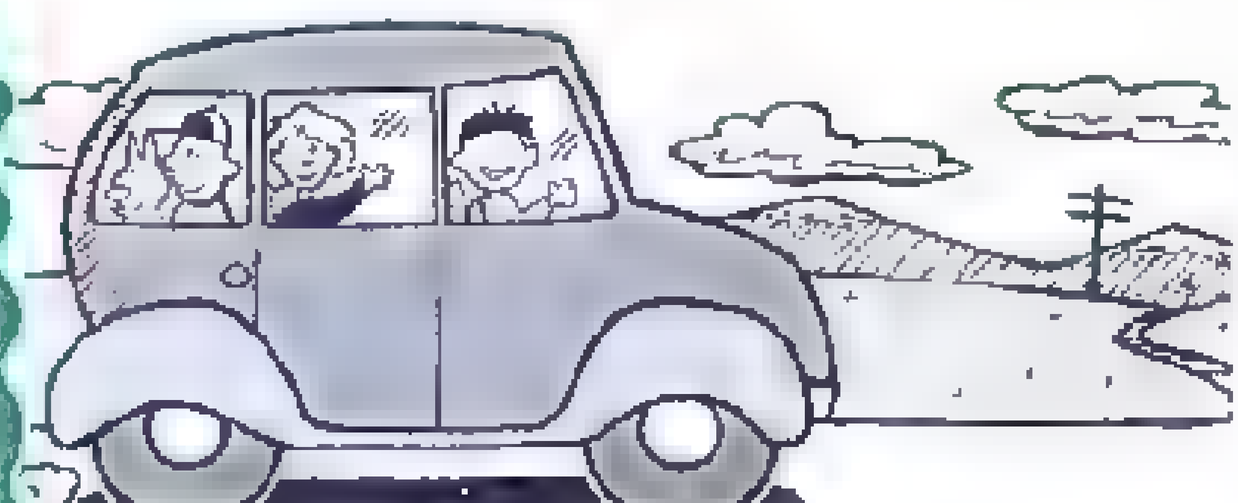
## NOT BY A LONG SHOT

How do we measure length and distance? With inches, feet, yards, miles—or with millimeters, centimeters, meters, and kilometers? Here is the conversion table.

| | |
|---|---|
| 1 mile = 5,280 feet<br>1 yard = 3 feet<br>1 foot = 12 inches | 1 kilometer = 1,000 meters<br>1 meter = 100 centimeters<br>1 centimeter = 10 millimeters |
| 1 mile = 1.61 kilometers<br>1 foot = 0.3048 meters<br>1 inch = 2.54 centimeters | 1 kilometer = 0.6214 miles<br>1 meter = 39.37 inches<br>1 millimeter = 0.0394 inches |

Q: Why did the metric chicken cross the road?

A: To see how many meters wide it was.



Answer the following questions and then fill in the answer key to find out which animal's tongue can grow up to 21 inches long—about as long as your arm. Slurp!

1. How long is a school bus?
   - f) 6 feet
   - g) 36 feet
   - h) 160 feet
   - i) 6 yards

2. How long is a twin bed?
   - h) 6 inches
   - i) 6 feet
   - j) 60 inches
   - k) 60 feet

3. How long is a football field?
   - r) 360 feet
   - s) 360 inches
   - t) 36 miles
   - u) 360 miles

4. How long is a dollar bill?
   - a) 6 inches
   - b) 60 centimeters
   - c) 6 feet
   - d) 6 millimeters

5. How long is a marathon?
   - c) 26.2 meters
   - d) 262 centimeters
   - e) 262 kilometers
   - f) 26.2 miles

6. How long is a blue whale?
   - c) 10 meters
   - d) 100 inches
   - e) 100 feet
   - f) 10 yards

| 1 | [illegible] | [illegible] | 4 | 5 | [illegible] | [illegible] |
|---|---|---|---|---|---|---|
| | | | | | | |

## Hink Pinks

Each silly answer in the following Hink Pinks is made up of two words that rhyme. HINT: The first six are one-syllable words; the last Hink Pink is made up of words that are two syllables each.

1. Pile of games that weighs 2,000 lbs ________________
2. 5,280 foot grin ________________
3. Two pastries that are each 12 inches long ________________
4. Very difficult 3 feet ________________
5. Rubber ball that weighs 16 ounces ________________
6. 28.4 grams that jumps quickly ________________
7. Urgent message that weighs 2.2 pounds ________________

### To St. Ives

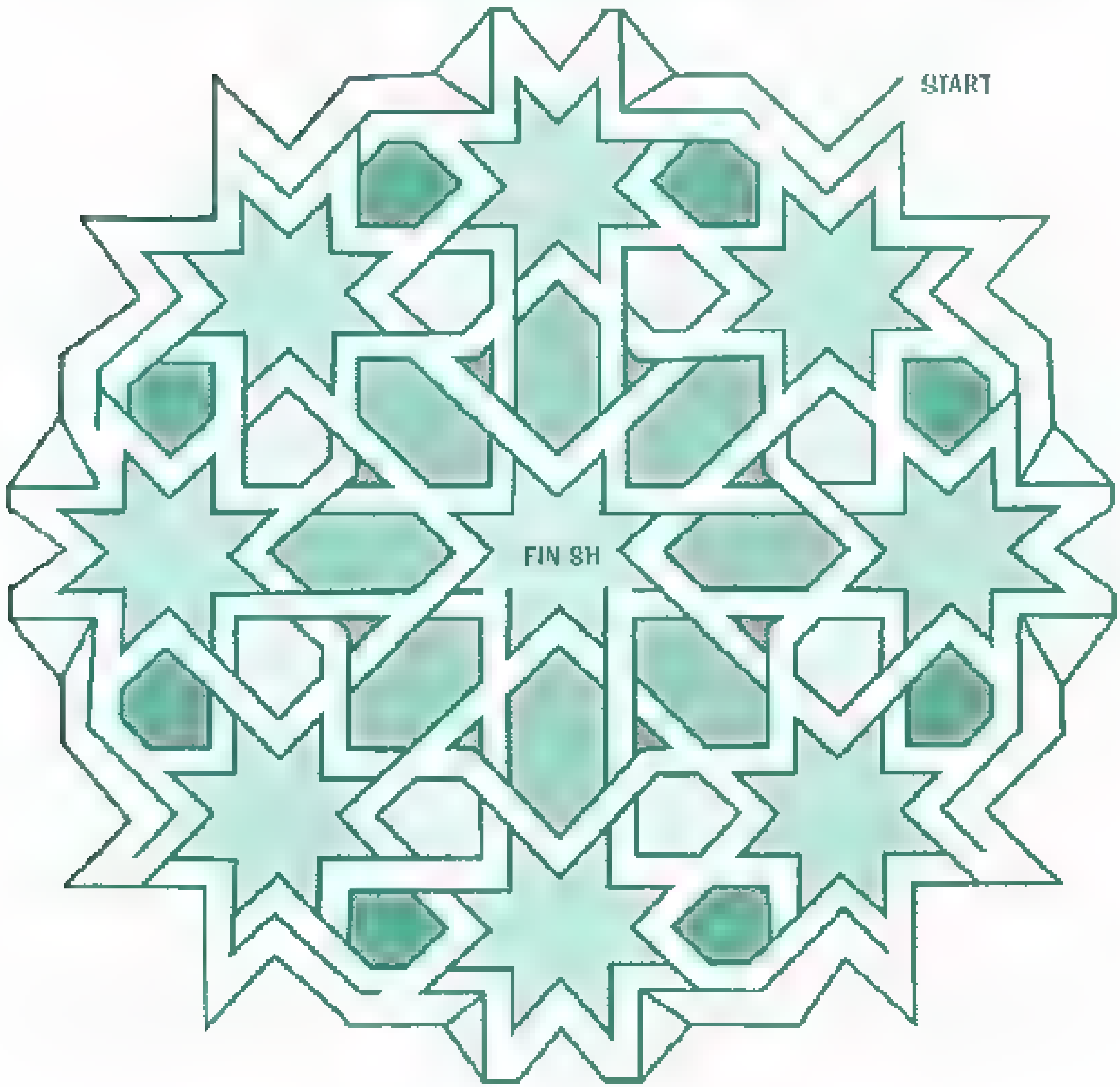
[illegible] to St. Ives
I met a [illegible]
Each who had six [illegible]
[illegible]
Each cat had four [illegible]
Each [illegible] had three mice,
each mouse had two pieces of cheese,
Each [illegible]
[illegible] were?

### Math Online

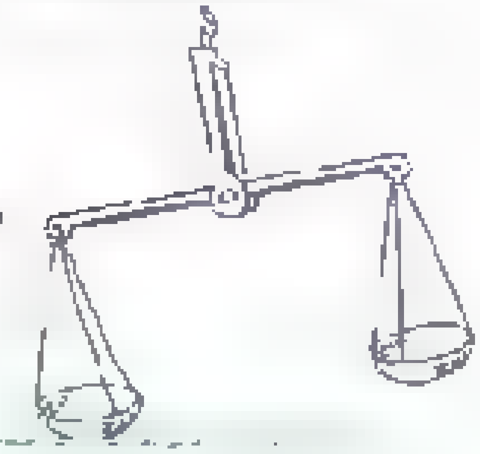
Journey across 42 powers of 10 at www.wordwizz.com/pwrsof10.htm. These pictures show one image at the submolecular level enlarged to 10 times as big, over and over until you are looking at the entire known universe. Choose the smallest image and then keep clicking on the +1 button to increase the zoom by a factor of 10.

# Star Power

Find the perfect center of this starry pattern.



## A WEIGHTY MATTER



It helps us to know how much things weigh. We measure candy in ounces, fruit and vegetables in pounds, and elephants in tons. Here are the conversions of metric and English weight measurements.

| | |
|---|---|
| 1 ton = 2,000 pounds<br>1 pound = 16 ounces | 1 kilogram = 1,000 grams<br>1 gram = 1,000 milligrams |
| 1 ton = 1,016.05 kilograms<br>1 pound = 0.45 kilograms<br>1 ounce = 28.4 grams | 1 kilogram = 2.2 pounds<br>1 gram = 0.035 ounces |

I am the world's largest animal. I can weigh 177 tons—that's 354,000 pounds. It would take more than 4,400 children averaging 80 pounds each to balance with me!

| | |
|---|---|
| 1. How much does an average man weigh?<br>a) 172 ounces<br>b) 172 pounds<br>c) 172 grams<br>d) 172 kilograms | 2. How much did a Tyrannosaurus rex weigh?<br>j) 60 kilograms<br>k) 60 pounds<br>l) 6 tons<br>m) 60 tons |

continued on facing page

continued from previous page

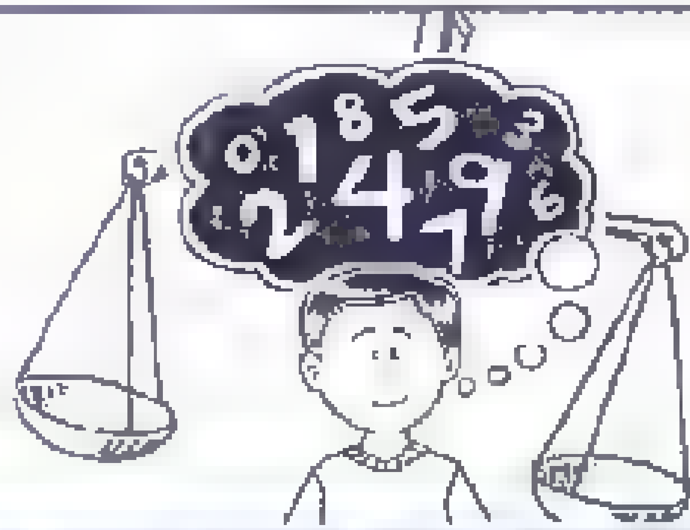
3. How much does an adult polar bear weigh?
   - u) 1,400 pounds
   - v) 140 pounds
   - w) 14 kilograms
   - x) 140 tons

6. How much does an ant weigh?
   - h) 4 milligrams
   - i) 4 kilograms
   - j) 4 ounces
   - k) 4 pounds

4. How much does this book weigh?
   - c) 11 grams
   - d) 11 milligrams
   - e) 11 ounces
   - f) 11 pounds

7. How much does an apple weigh?
   - a) 160 grams
   - b) 160 ounces
   - c) 16 pounds
   - d) 16 grams

5. How much does a nickel weigh?
   - w) 5 grams
   - x) 0.05 ounces
   - y) 50 ounces
   - z) 50 grams

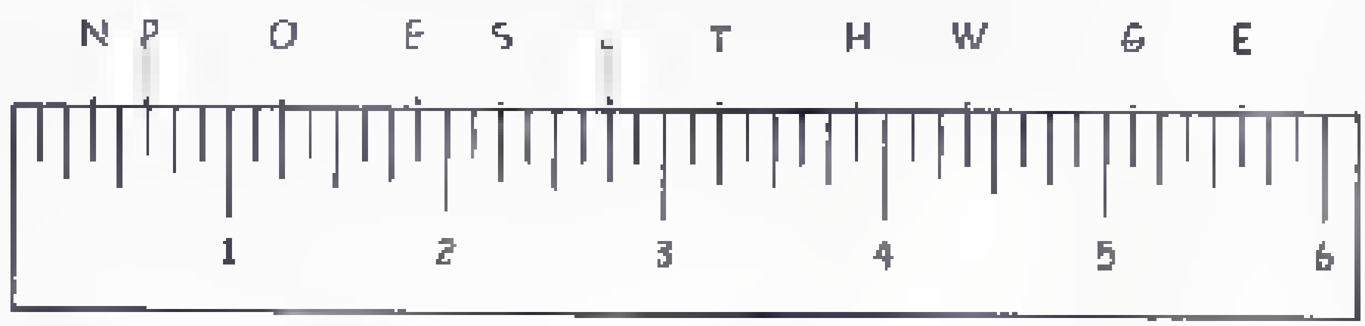


| 1 | 2 | 3 | 4 | | 5 | 6 | 7 | 2 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |

# Measuring Your ZZZs

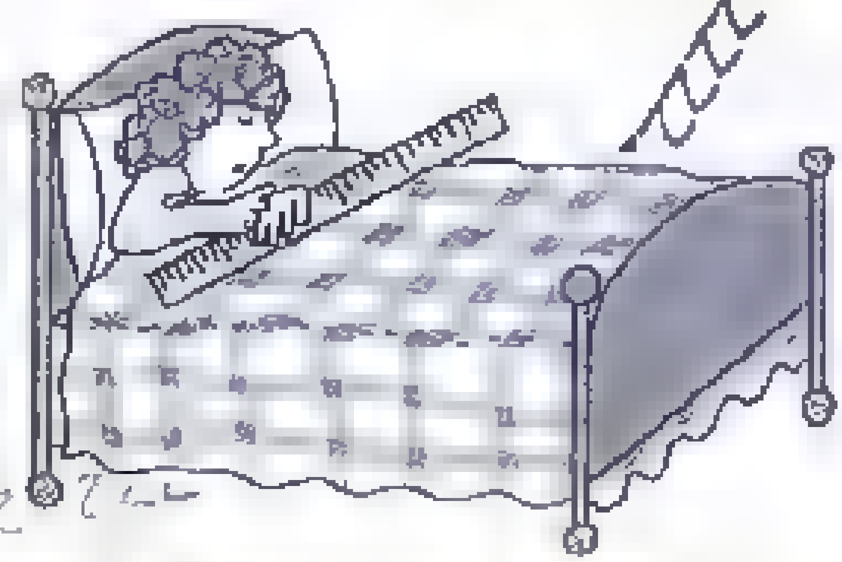
Write each letter in the space above the measurement that matches its place on the ruler. When you're finished, you will have the answer to the following riddle:

**Why did the math teacher take a nap with a ruler?**




$3\frac{2}{8}$ 1.25   2.25 $1\frac{7}{8}$ $5\frac{5}{8}$   $3\frac{7}{8}$ 1.25 $4\frac{3}{8}$   2.75 $1\frac{1}{4}$ $\frac{3}{8}$ $5\frac{1}{8}$

$2\frac{1}{4}$ $3\frac{7}{8}$ $1\frac{7}{8}$   $2\frac{2}{8}$ $2\frac{3}{4}$ $1\frac{7}{8}$ $\frac{5}{8}$ $3\frac{2}{8}$



0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

## STACKING UP ZEROS

Why is the [illegible] system based on 10? Because our entire number system is built on multiples of 10. Think of it this way. You've got 1, 2, 3, 4, 5, 6, 7, 8, 9 and then you go back to 1 and just add a 0 to it = 10. Add another 0 and you get 100. How many zeros can you keep on stacking? Take a look here.

| NUMBER OF ZEROS | WHAT YOU GET |
|---|---|
| 0 | one |
| 1 | ten |
| 2 | hundred |
| 3 | thousand |
| 4 | ten thousand |
| 5 | hundred thousand |
| 6 | million |
| 7 | ten million |
| 8 | hundred million |
| 9 | billion |
| 12 | trillion |
| 15 | quadrillion |
| 100 | googol |
| googol | googolplex |

### Lost Billions

It looks like there are billions and billions of the word BILLIONS in this grid, but it only appears correctly spelled one time. It could be left to right, up and down, diagonal, or even backward.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| B | I | L | L | I | O | N | N | S | B | B | I |
| B | I | L | S | B | S | N | O | I | L | I | B |
| I | B | S | S | N | B | I | L | L | O | N | S |
| L | S | N | N | B | O | B | I | L | L | N | O |
| L | N | I | O | I | I | I | B | B | O | B | I |
| L | O | O | I | L | B | O | L | I | B | I | L |
| I | I | L | L | O | B | S | I | L | O | B | I |
| O | L | L | L | O | N | L | B | O | I | S | L |
| N | L | I | B | I | L | I | O | B | B | I | I |
| S | I | B | L | I | B | O | B | I | L | I | B |
| I | B | L | B | B | I | L | O | O | N | S | I |
| O | B | B | I | L | I | O | O | N | S | O | L |

## Googols of Fun

A googol is a REALLY big number. If a ten has one zero (0), how many zeros do you think a googol has? Using a simple number substitution (A=1, B=2, C=3, etc.), see if you can break this number code.

1 7•15•15•7•15•12 8•1•19

_ _ _ _ _ _ _ _ _ _

15•14•5 8•21•14•4•18•5•4

_ _ _ _ _ _ _ _ _ _

26•5•18•15•19

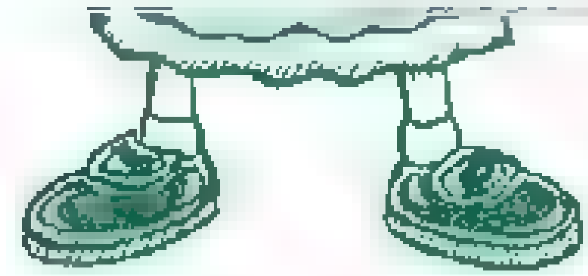
_ _ _ _ _

### What's Beyond Googolplex?

The rules of math say that you can keep counting for infinity. That means, numbers keep going and going, and there is no such thing as the last or highest number. Infinity is not a number—it's a concept that something is never-ending. Mathematicians use ∞ to represent infinity in mathematical calculations.

#### Counting Forever

The symbol ∞ has been around for more than two thousand years. The Romans used it to represent 1,000, a BIG number to them. Around the year 1650, an English mathematician, John Wallis, proposed that this symbol ∞ be used to represent infinity, and we have been doing just that ever since.



# On Your Toes

*Use this fraction code to figure out the riddle*

**CODE**

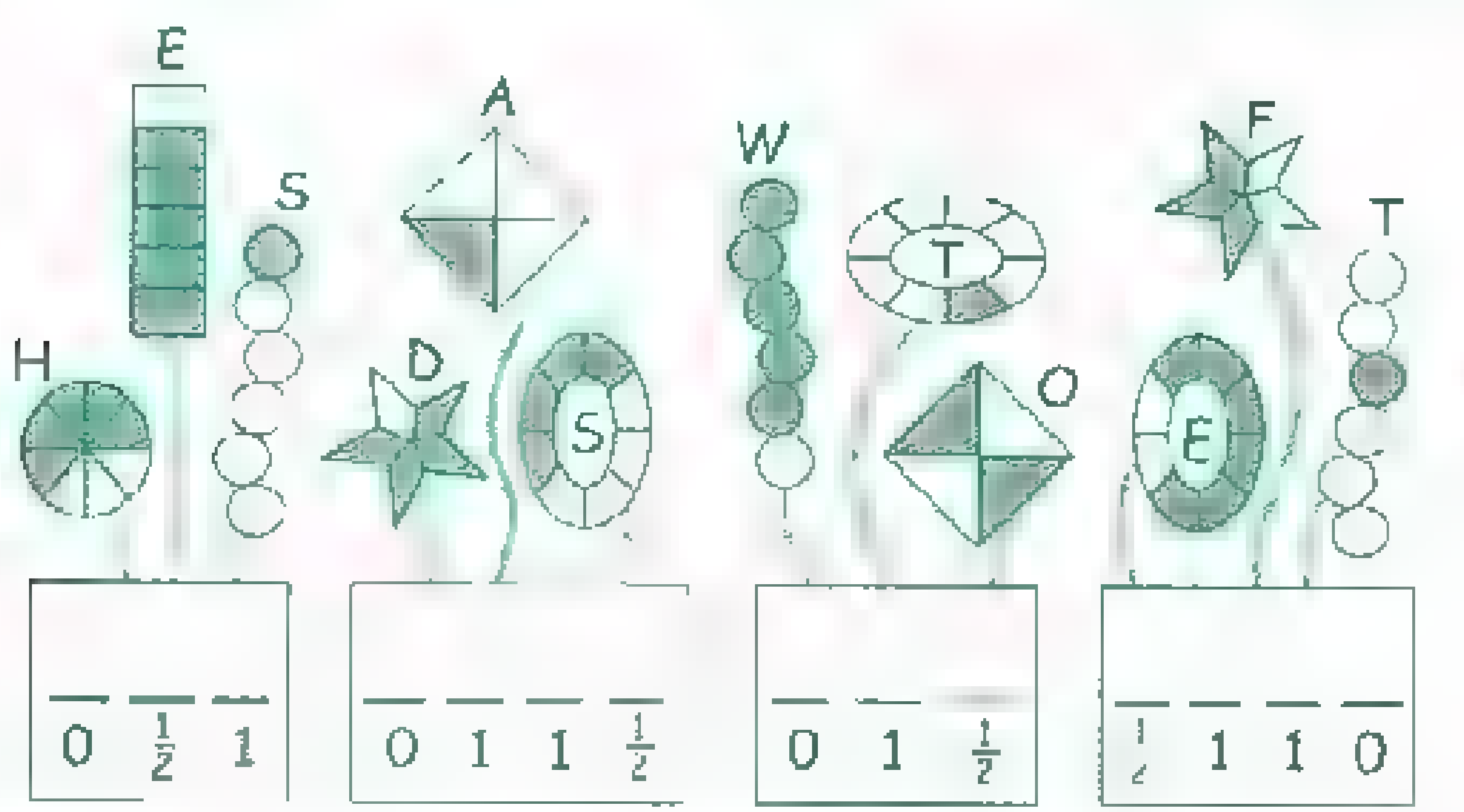
The white part of each shape is empty. The shaded part of each shape is filled. Estimate how much of each shape is filled using the following rules:

If the shape is almost empty, call it 0.
If the shape is almost full, call it 1.
If the shape is between full and empty, call it ½.

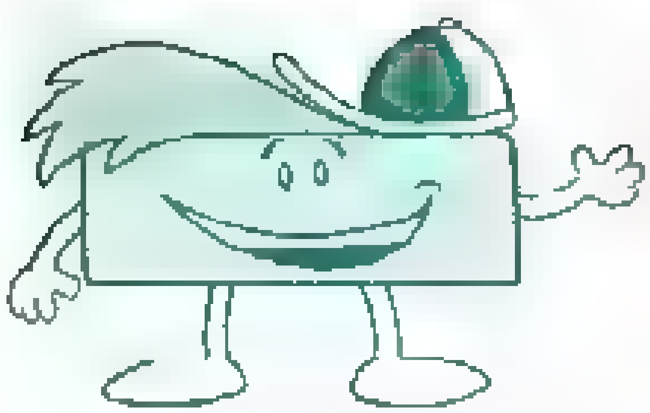
**HOW TO USE THE CODE**

Look at the fraction or number below each blank. Using the shapes connected to each word box, pick the shape that is closest to that fraction or number. Write the letter of that shape on the blank.

**RIDDLE: How does a math student make her shoes longer?**

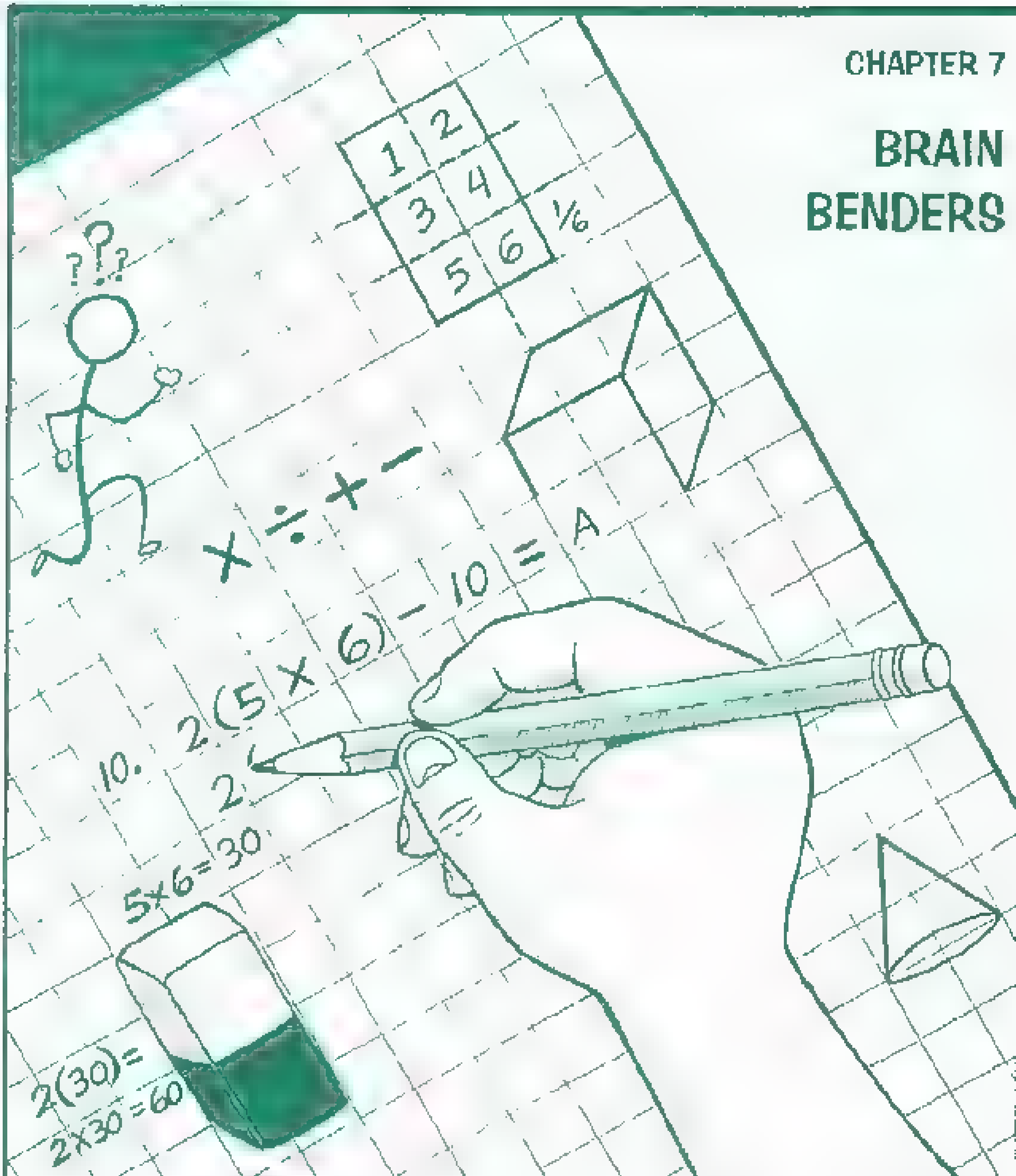


# NOTES



CHAPTER 7

# BRAIN BENDERS



## LOGICAL MATH

Most of us, when we think of math, think of numbers. But math is more than just numbers. You've already experienced some geometry, [illegible] ges. Another area of math that does not have to involve numbers is logic.

### FUN FACT

**Just Like Sherlock Holmes**

The fictional detective Sherlock Holmes is famous for using logic to solve cases based on clues he had observed. Logic and reasoning allow us to figure things out using known facts or clues instead of simply guessing. All of us use logic every day of our lives, although some people are better at it than others.

### Go Figure!

1. You have two coins. Added together, they equal 15 cents—but one is not a nickel. How is that possible?

2. Can you take four nines (9, 9, 9, 9) and arrange them to make an equation that totals 100? You can use any math function, but you can only use each nine once.

3. What do you get if you divide 30 by half and add 10?

4. If there are five apples and you take away three, how many apples do you have?

5. What is the meaning of FaπCE?

6. What is the meaning of MILONELION?

7. What is the meaning of this phrase: ONE, ONE, ONE, ONE, ONE, ONE, ANOTHER, ANOTHER, ANOTHER, ANOTHER, ANOTHER, ANOTHER?

8. A clerk in a butcher shop is 6' 2" tall. What does he weigh?

# Moving Around

See if you can use logic to solve the puzzle about four fast friends: Gary, Harry, Larry and Mary who lived in Los Angeles. One day, the four friends got news that they would be moving to four different cities: Atlanta, Boston, Chicago, and Dallas. By listening to their parents talk, the friends learned the following:

* Gary's family was moving to either Boston or Dallas.
* Harry's family was not moving to either Boston or Chicago.
* Either Harry or Mary was moving to Atlanta.
* If Harry moves to Atlanta, then Gary moves to Chicago.
* If Larry moves to Chicago, then Gary does not move to Dallas.

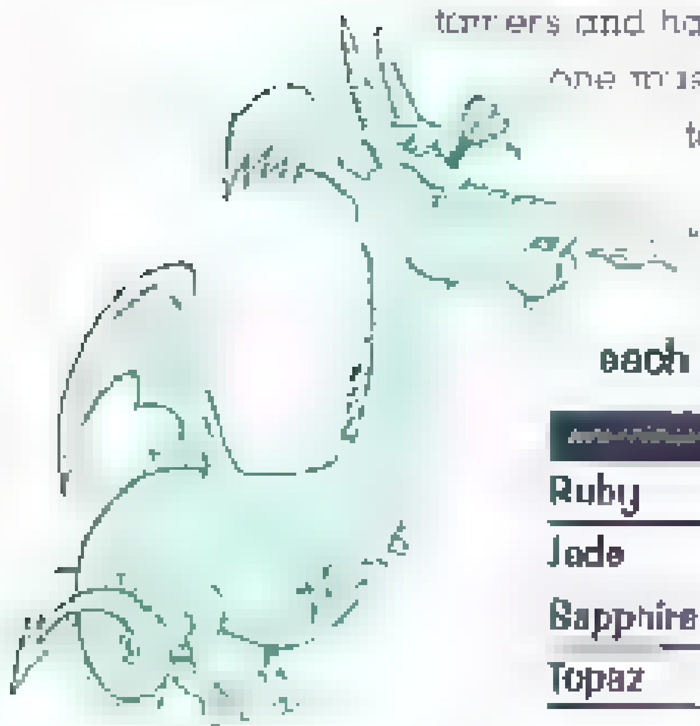
Who is moving where? Use the table to figure out the answer.

| | ATLANTA | BOSTON | CHICAGO | DALLAS |
|---|---|---|---|---|
| Gary | | | | |
| Harry | | | | |
| Larry | | | | |
| Mary | | | | |

## IN THE ENCHANTED KINGDOM

The logic puzzles on this page take place in an enchanted kingdom, at a castle [illegible] princesses — Ruby, Sapphire, Jade, and Topaz — as well as their mother, Queen [illegible], and their four pet dragons.

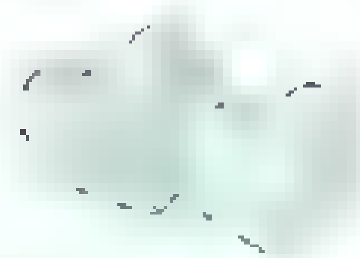
### Taming Dragons



These princesses don't need anyone to rescue them — they are dragon tamers and have four days to tame their [illegible] dragons. Every day each one must work with a different dragon. On the **second day**, Ruby tamed Spitfire and Sapphire tamed Forktail. On the **third day**, Jade tamed Blaze and Topaz tamed Smokey. On the **fourth day**, Sapphire tamed Smokey and Topaz tamed Forktail. **Which dragon did each princess tame on each day?**

| | DAY 1 | DAY 2 | DAY 3 | DAY 4 |
|---|---|---|---|---|
| Ruby | | | | |
| Jade | | | | |
| Sapphire | | | | |
| Topaz | | | | |

### The Marriage Proposa



[illegible] Prince Pyr[illegible] **black** [illegible] **white** [illegible]

[illegible] followed him and saw him put two black pebbles into the purse. At first, she's dismayed, but then gets an idea. **How can Ruby trick the prince?**

## IN THE LAND OF CONFUSION

A[illegible] Land of Confusion are either sages [illegible] **Sages** are noted for their honesty [illegible] the truth. **Jesters**, however, [illegible] around and never tell the truth. Unfortunately, there [illegible] here.

[illegible] the Enchanted Kingdom and [illegible] to the Land of Confusion. [illegible] three men, Xavier, Yale, and Zachary, [illegible] How [illegible] are sages?

**Xavier says that he is not a jester.**
**Yale says that Xavier is lying.**
**Zachary adds, "Yale is lying."**

### Can you figure out who is a sage and who is a jester?

### Two Kids in the Land of Confusion

[illegible] on walking. Just as she app[illegible] and a girl sitting on the steps, and [illegible] statements:

1. "I'm a boy," says the child with black hair.
2. "I'm a girl," says the child with red hair.
3. "Exactly one of us is a sage," says the boy.

### Are the boy and girl sages and/or jesters, and what color hair does each one have?

[illegible]

## A Boast?

Upon entering the emperor's palace, Ima overheard a man say, "[illegible] am a sage, then so is my son."

**What can you tell about him or his son?**

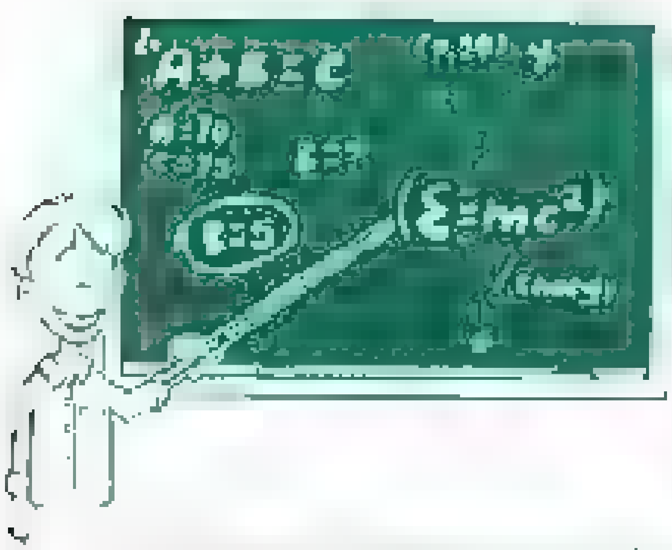
You will find the answers in the Puzzle Answers section, page 131.

## Meeting the King

Inside the palace, Ima went looking for the King, and she met some sages. She asked them if the King is a sage or a jester, and they told her that they didn't know, but that according to their laws, he must be a citizen of the Land of Confusion, either a sage or a jester.

Just then, a man entered the hall wearing a crown and a beautiful purple cloak. The sages told Ima that it was the King, and that she could ask him whether he was a sage or a jester. To her question, the King responded, "I am a jester."

**Immediately, the sages cried: "Imposter! Arrest that man!" Why?**




### Einstein's Math Troubles

*"Do not worry too much about your difficulties in mathematics, I can assure you mine are still greater."*

*—Albert Einstein*

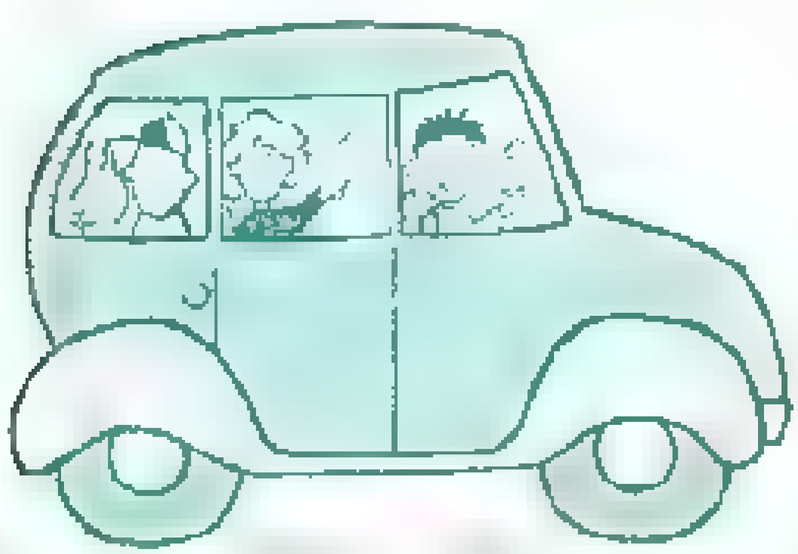
### Math Online

For fun math games, check out www.mathspower.org. In particular, see if you can solve the code-breaker puzzle.

# Read the Numbers

Numbers are used for counting, but did you know they can also be used for spelling? Numbers like 4, 8, and 2 sound the same as familiar words we use all the time. People who want to write short messages often substitute a single number for a group of letters. For example "L8" spells "late," but uses only two characters instead of four. This kind of number spelling is often used on license plates, where there is room for only six or seven characters. **See if you can match the following people with their fancy license plates:**

1. Musician who plays a big horn.
2. A person who is always lucky.
3. Someone who is very polite.
4. A person who is always happy
5. Someone who can predict the weather.
6. A person who is hardly ever on time.
7. ~~Artist.~~
8. Someone who is impatient.
9. A person with a bad memory.
10. Someone who likes surprises.
11. A frequent visitor to a skating rink.
12. Someone who likes a particular sport



| | | |
|---|---|---|
| CR8TV | 1DRFL | 42N8 |
| 7 CREATIVE | | |
| 4EVRL8 | I4GOT | 1OSNY1 |
| H82W8 | 2BA | W8NC |
| 4CAST | 10Q | SK8R |

## Simple Symbols

[illegible] equals one [illegible] the possible [illegible] the numbers for the [illegible] the subtraction problems [illegible]

**HINT:** [illegible] numbers in the [illegible]

Possible numbers: 1 2 4 6

**Q: Where's the best place to hide if you're scared?**

**A: Inside a math book—because there's safety in numbers.**

### A Card Game

Two kids play five games of gin rummy. Each kid wins the same number of games, but there are no tied games. How is this possible?

The two kids weren't playing each other.

### Buying Numbers

This conversation doesn't add up. Can you tell what's for sale?

**Customer:** How much is 1?

**Salesman:** 30 cents.

**Customer:** I'd like 14, please.

**Salesman:** That will be 60 cents.

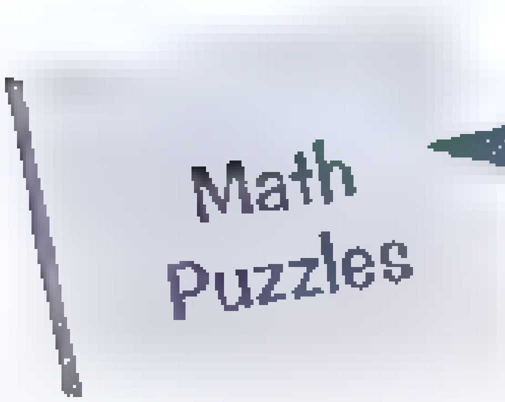
**Customer:** Oops, I really need 114.

**Salesman:** No problem. That will be 90 cents.

# I Can't Find It!

The numbers 1, 2, 3, 4, and 5 appear in a row only one time in this number grid! The answer can be up, down, side to side, or diagonal. Look extra carefully—the numbers might also appear backward!

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 2 | 1 | 3 | 1 | 2 | 3 | 5 |
| 2 | 2 | 3 | 3 | 1 | 2 | 1 | 4 | 1 |
| 1 | 2 | 1 | 2 | 5 | 2 | 2 | 1 | 2 |
| 2 | 3 | 1 | 4 | 3 | 1 | 4 | 2 | 5 |
| 1 | 2 | 3 | 4 | 2 | 3 | 3 | 4 | 4 |
| 4 | 1 | 5 | 2 | 1 | 2 | 1 | 3 | 3 |
| 5 | 3 | 4 | 1 | 2 | 4 | 4 | 5 | 5 |
| 1 | 2 | 2 | 1 | 3 | 4 | 5 | 1 | 2 |
| 3 | 1 | 1 | 2 | 4 | 3 | 3 | 2 | 1 |



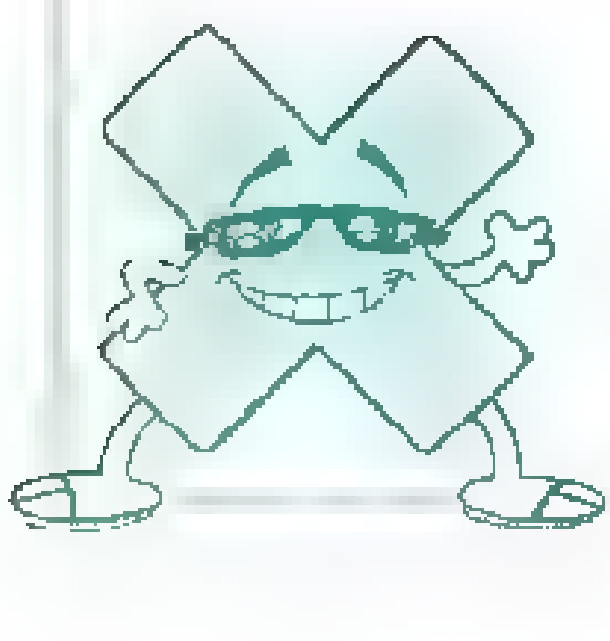

# I Found It!

Mr. Math asked four of his students to see how many math puzzle books they could find in the public library. How many books did each student find? Use the clues to complete the chart.

| Students | # of books each student found |
|---|---|
| Jasmine | |
| Katlyn | |
| Josh | |
| Ethan | |
| TOTAL number of puzzle books found | |

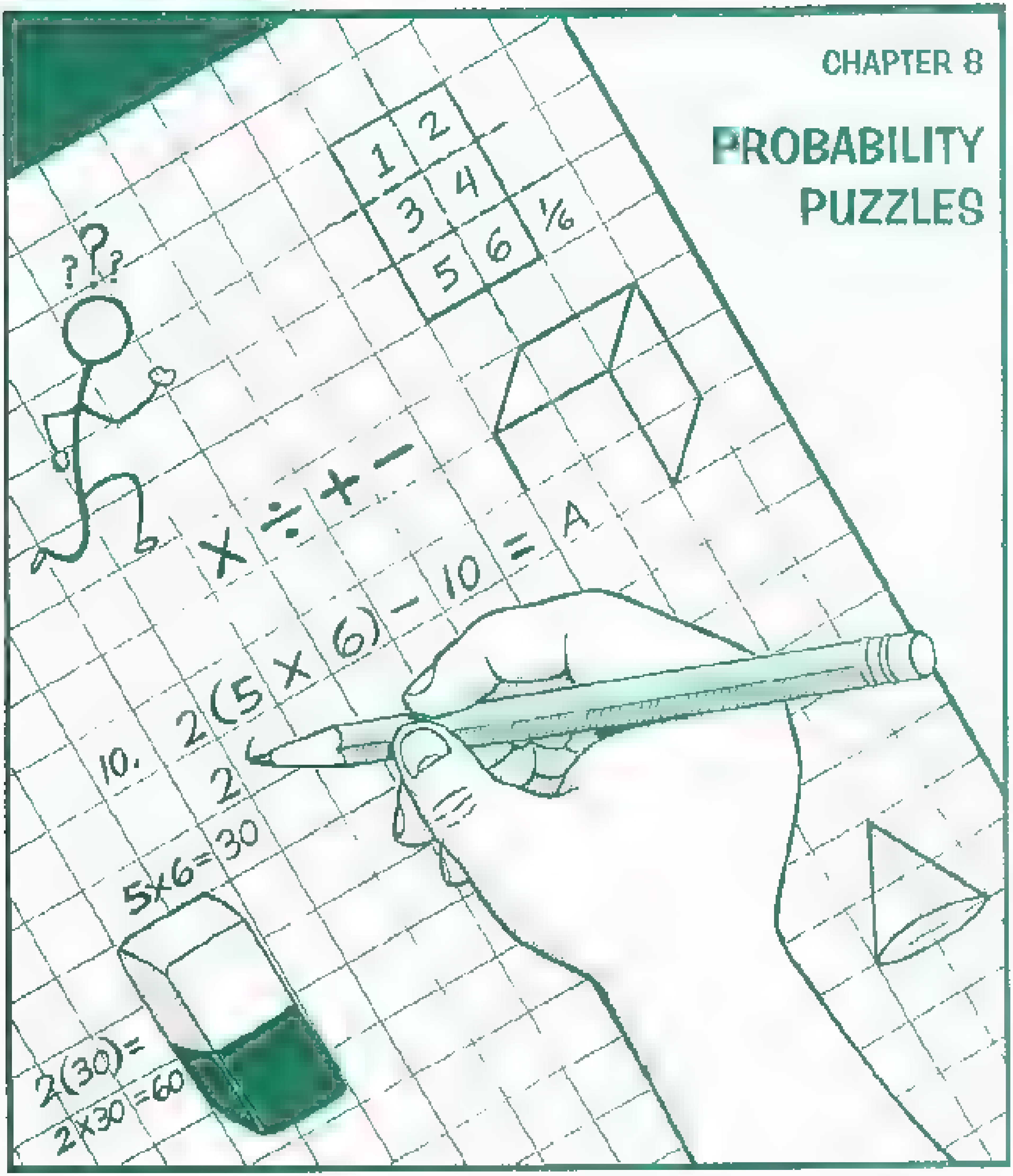
1. The four students found a total of 13 math puzzle books.
2. None of the students found more than 5 puzzle books.
3. None of the children found the same number of puzzle books.
4. Jasmine found 3 puzzle books.
5. Katlyn found fewer puzzle books than Jasmine.
6. Ethan found the most puzzle books.

# NOTES



CHAPTER 8

# PROBABILITY PUZZLES



## A PROBABLE CAUSE

You might be surprised to know that probability is a relatively new area of math. P[illegible] began to be studied in the mid-1600s. That may seem like a long time ago, but remember [illegible] people have been doing geometry for thousands of [illegible].

**Probability** is invol[illegible] any time we do something where we can't know in advance what is going to happen. When we toss a coin in the air, we know it is going to come back down. However, we don't know if it will land as heads or tails. That is probability.

### WORDS to KNOW

***probability:*** The chances of something happening—actually, the ratio of the number of times it could happen compared to the number of possible things that could happen. For example, when you throw dice, you have 1/6 chances of getting each particular number.

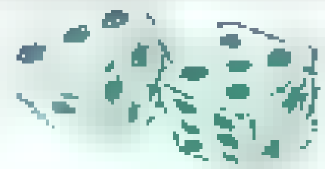
### Probability Scrabble

**Can you guess how many other words you could make by using any of the eleven letters in the word PROBABILITY?** Do you think you [illegible] number closer to [illegible] you can use each [illegible] as many times as it appears in PROBABILITY. For example, you could spell BABY because there are two Bs, but you can't spell ROOT, because there is only one O.

**A Coin Toss**

**The probability of getting a head or a tail on any coin toss is ½ heads and ½ tails. Do a simulated coin-toss experiment at *www.ucs.louisiana.edu/faculty/bllim/java/pregeinnotes/CoinToss.html*. First, try 25 tosses, then 100, and then 500. When we tried it, we got 10/15, 50/50, and 251/249.**

## PLAYING DICE

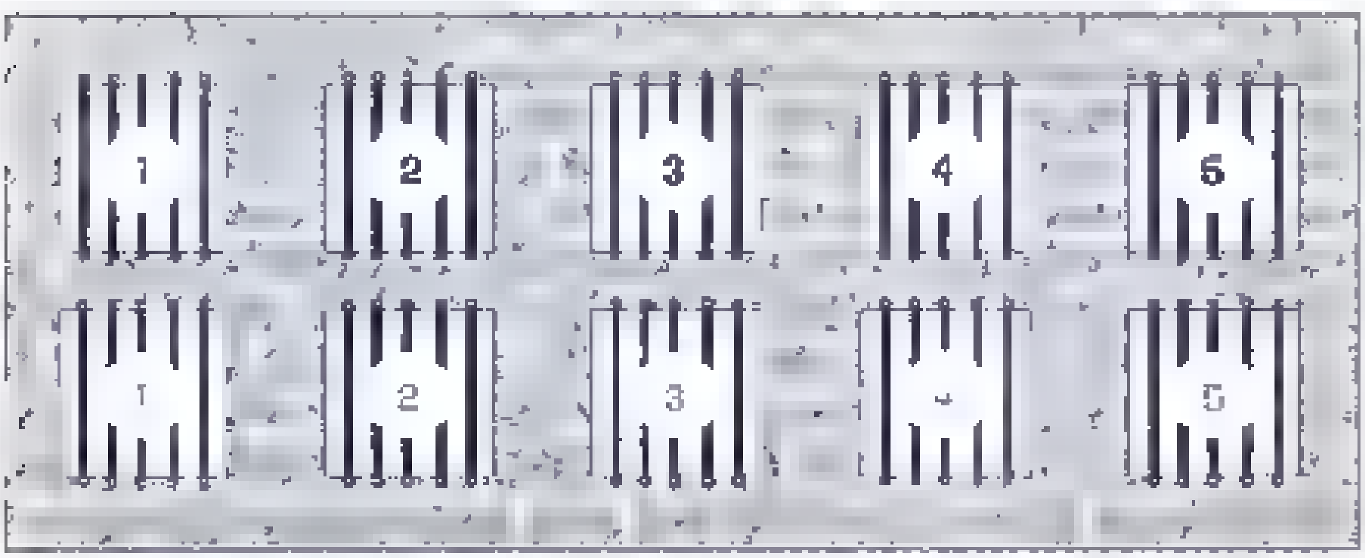


When you roll the die, you are playing with probability. With each roll, you are as likely to get 1 as you are 2, 3, 4, 5, or 6. That means that you have one in six chances to get any number. And each time you throw the die, the probability of getting a particular number remains the same.

### What If You Roll Two Dice?

Can you figure out what your probabilities are of getting a 1, 2, 3, 4, 5, or 6 if you roll two dice? Will your chances increase or decrease?

## Prisoners Game




This is a game for two players. To play, you will need the following:

- **Five pennies and five nickels (or two sets of other markers)**
- **A pair of dice**
- **Paper and pencil for keeping score**

Players should each have five markers, which are prisoners. Player 1 places his or her five prisoners in each cell of the top row. Player 2 places prisoners in the bottom row. Players take turns rolling the dice and subtracting the smaller number from the larger. If the difference matches a cell number, the prisoner kept in that cell goes free (is taken off the board). **Whoever frees all the prisoners first, wins!**

### A Variation on the Prisoners Game

This time, each player picks where to put his or her prisoners. For example, Player 1 could put a prisoner in cells 1, 2, and 3 of the top row and cells 3, 4, and 5 of the bottom row. Each roll of the dice frees two prisoners at a time. After playing several rounds, who has the best strategy?

## Who Owns That Car?

[illegible] man came into the [illegible] his new car. He [illegible] [illegible] [illegible]

**What was the man's name?**


# Let 'er Roll!

Unlike the Prisoners Game, this one offers equal chances of winning to all the players, and you can have as many as six people join the game. Here is what you will need to play:

- **Piece of paper and pencil for each player**
- **Pair of dice**

You also need to make a game card for each player. To do that, draw a five-squares-by-five-squares grid. You can use a ruler to make the lines straight, but it doesn't matter if they're [illegible]

Then, have one player read the following list of numbers:

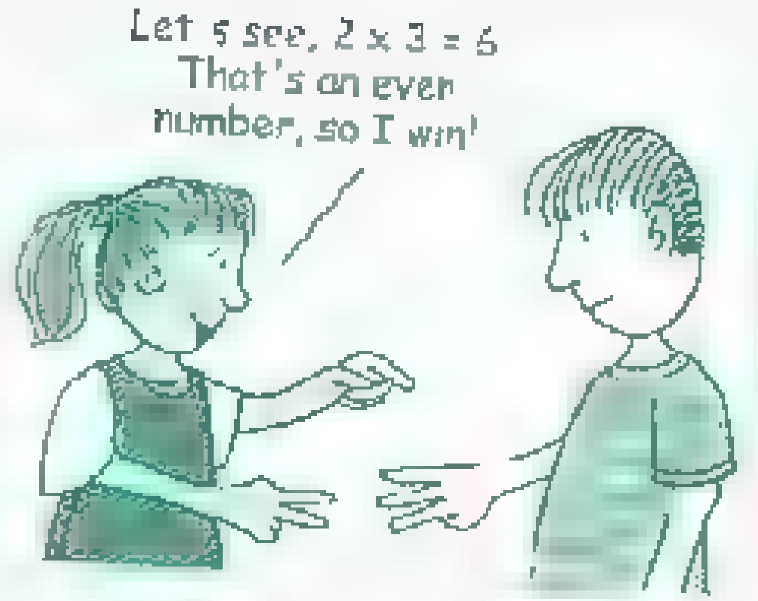
**1, 36, 9, 24, 18, 8, 6, 15, 30, 25, 10, 24, 18, 6, 3, 12, 2, 4, 12, 16, 9, 12, 16, 9, 12, 20, 6, 10**

As each number is called out, each player should write it down in any one of the squares in the grid, until all the spaces are filled. Some numbers will appear twice.

Once you are done, the game can begin. Players take turn rolling the two dice, multiplying the two results, and then covering one number on the grid. **The first one to cover five squares in a row (in any direction) is the winner.**

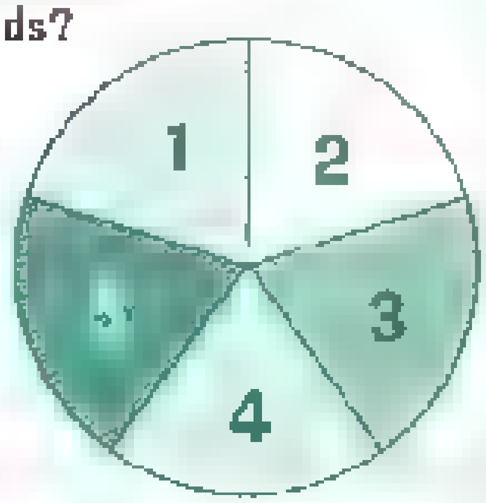
# Even and Odd

Here is a game that uses a principle similar to the Prisoners Game and Letter Roll. To play, you will need two players, **Player E** (even) and **Player O** (odd), plus the following:




- **Piece of paper**
- **Pencil**
- **Paper clip**

On the count of three, both players show each other one to five fingers on one hand. Multiply the number of fingers showing on one player's hand by the number of fingers showing on the other player's hand. If the product is even, Player E wins; if the product is odd, Player O wins. Keep a tally of twenty rounds. **Which player wins the most rounds?**




You should have noticed that Player E won many more games than Player O. Will Player E still win if both players use a spinner instead of their fingers? Try it. Use a pencil to hold a paper clip in the center of the spinner. Flick the paper clip around the spinner. Keep a tally of twenty rounds. **Now, which player won the most games?**

Space
for
Tallies

(see the following page for possible explanations)

## Even and Odd—Explained

What's going on? Why does Player E keep winning? There are two possible explanations.

**1.** If Player F [illegible] either two or four fingers, the answer [illegible] even regardless [illegible] the number of fingers Player [illegible] shows. Player F [illegible]

**2.** [illegible] Player E [illegible] fingers [illegible] the same as using the spinner. Player E [illegible] the time. To see [illegible], **fill in this multiplication table** and [illegible] the even numbers. [illegible]

| × | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | | |
| 2 | 2 | 4 | | | |
| 3 | 3 | | | | |
| 4 | | | | | |
| 5 | | | | | |

▲When you look at the results, it's easy to see which kind of number is more likely to win!

## Get out of Here

Make [illegible] from START to END by following a [illegible] path [illegible] even, odd, even, odd, etc.

How many different paths do you think you might start before you find the correct one?

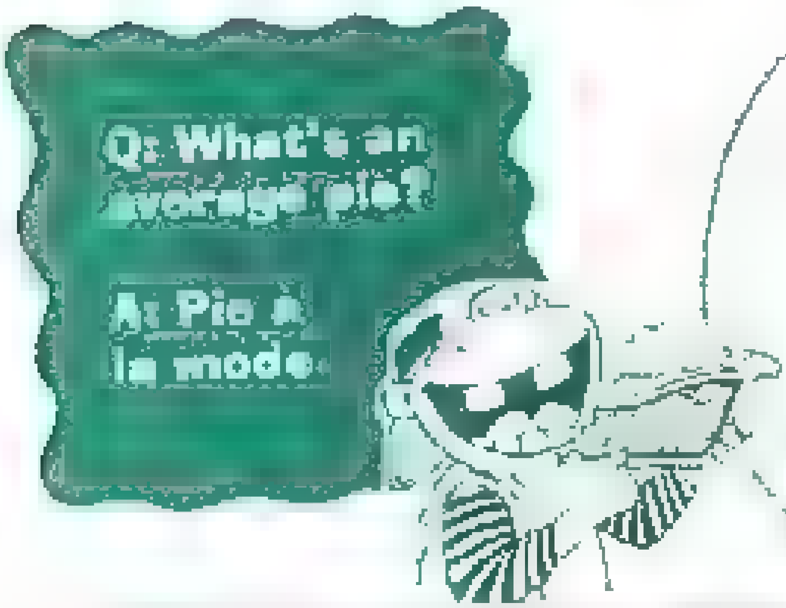
| | | | | | |
|---|---|---|---|---|---|
| START 2 | 7 | 4 | 3 | 12 | 7 |
| 3 | 6 | 3 | 8 | 11 | 2 |
| 6 | 9 | 4 | 13 | 4 | 15 |
| 3 | 22 | 5 | 12 | 11 | 22 |
| 12 | 17 | 2 | 9 | 20 | 5 END |

# An Average Day

What do you know about estimating averages? Each blank balloon represents an average value. Fill in the following story with your best estimates!

I'm just an average ten-year-old who is ______ inches tall. I am in school for ______ hours per year. After school, I eat a snack. Usually it's popcorn. One popped kernel weighs about ______ grams and there are about ______ popped kernels in one cup. Today, we are out of popcorn, so I eat ice cream. My favorite flavor is ______. I watch a half-hour TV show, but there is an average of ______ commercials every 30 minutes, so the show is really only ______ minutes long. Before I go to bed, I still check under the bed for monsters. After all, there is an average of ______ monsters per bed.

▶ One of the averages above is a mode. Which one is it? ____________________




## WORDS to KNOW

**averages:** There are three ways to calculate the average. The **mean** is what you get if you add up all the values and divide the total by the number of values. This is typically what people mean when they use the word **average**. The **median** of a set of values is the value in the middle of the set when the set is written in order. The **mode** of a set of values is the value that occurs most often. Mode is usually used as the average when the values are not numbers, like eye or hair color.

# Hidden Numbers, Etc.

There are ten sentences hidden in this grid, and only one sentence per line, but here's the trick: Within each sentence are letters that spell out a number ONE through TEN. In the grid, though, the spelled-out numbers have been replaced by the corresponding

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| B | U | T | T | E | R | H | Y | T | F | L | Y | T | O | P | 7 | 3 | Y | R | E | S |
| T | H | E | 2 | I | O | F | 1 | 0 | E | A | T | H | E | R | E | W | E | 9 | S | H |
| B | E | P | L | T | A | P | E | 9 | A | C | H | B | A | G | W | H | E | T | 2 | M |
| W | H | E | N | I | 8 | A | N | Y | T | R | E | B | M | K | 9 | 0 | 4 | V | A | T |
| 7 | O | U | R | W | 8 | H | E | A | P | P | L | E | S | P | L | E | A | S | E | 8 |
| I | T | I | 7 | N | O | T | O | D | D | I | F | 1 | C | B | 2 | B | N | 3 | M | O |
| B | O | P | N | E | S | 5 | 9 | Y | S | E | E | I | 5 | G | O | T | M | A | I | L |
| M | Y | C | A | T | W | H | A | T | I | 4 | P | H | 1 | R | I | N | G | S | R | I |
| W | H | Y | 7 | J | I | M | L | P | O | T | T | E | R | S | R | E | W | 5 | D | U |
| T | T | H | E | P | O | N | D | I | S | F | I | L | L | E | D | W | I | 3 | D | S |
| V | I | T | A | M | I | 4 | T | H | A | 2 | U | L | D | B | E | N | I | C | E | K |
| N | Y | T | H | R | T | H | I | S | I | S | A | 4 | A | S | T | Y | J | I | O | B |
| 6 | T | H | E | S | H | E | I | S | T | O | 1 | A | R | T | H | E | E | D | G | E |
| H | E | H | A | S | A | B | A | R | 9 | A | S | T | C | O | N | C | O | R | D | H |
| B | U | T | I | 6 | D | O | N | 2 | R | R | Y | B | E | H | A | P | P | Y | P | Y |

(Continued)

digits 1 through 10. Circle the sentences as you find them and write them out with correct spelling, and don't forget the punctuation! HINT: The sentences are all horizontal, but not every line has a hidden sentence in it.

SAMPLE: I often eat here.

1

2

3.

4

5

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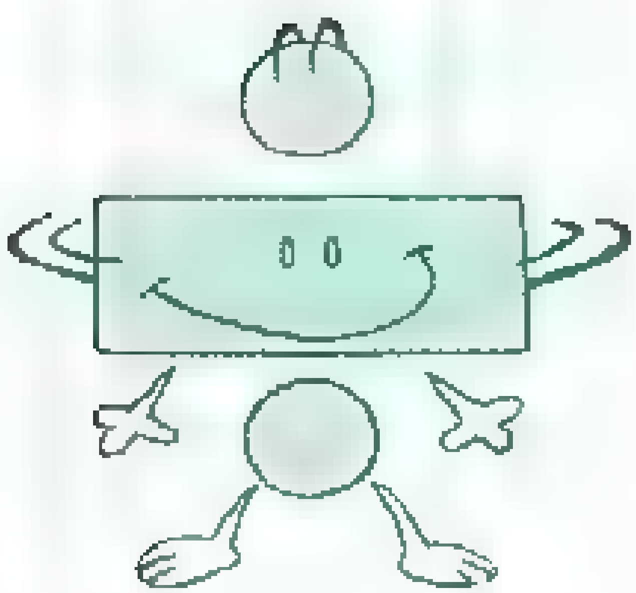
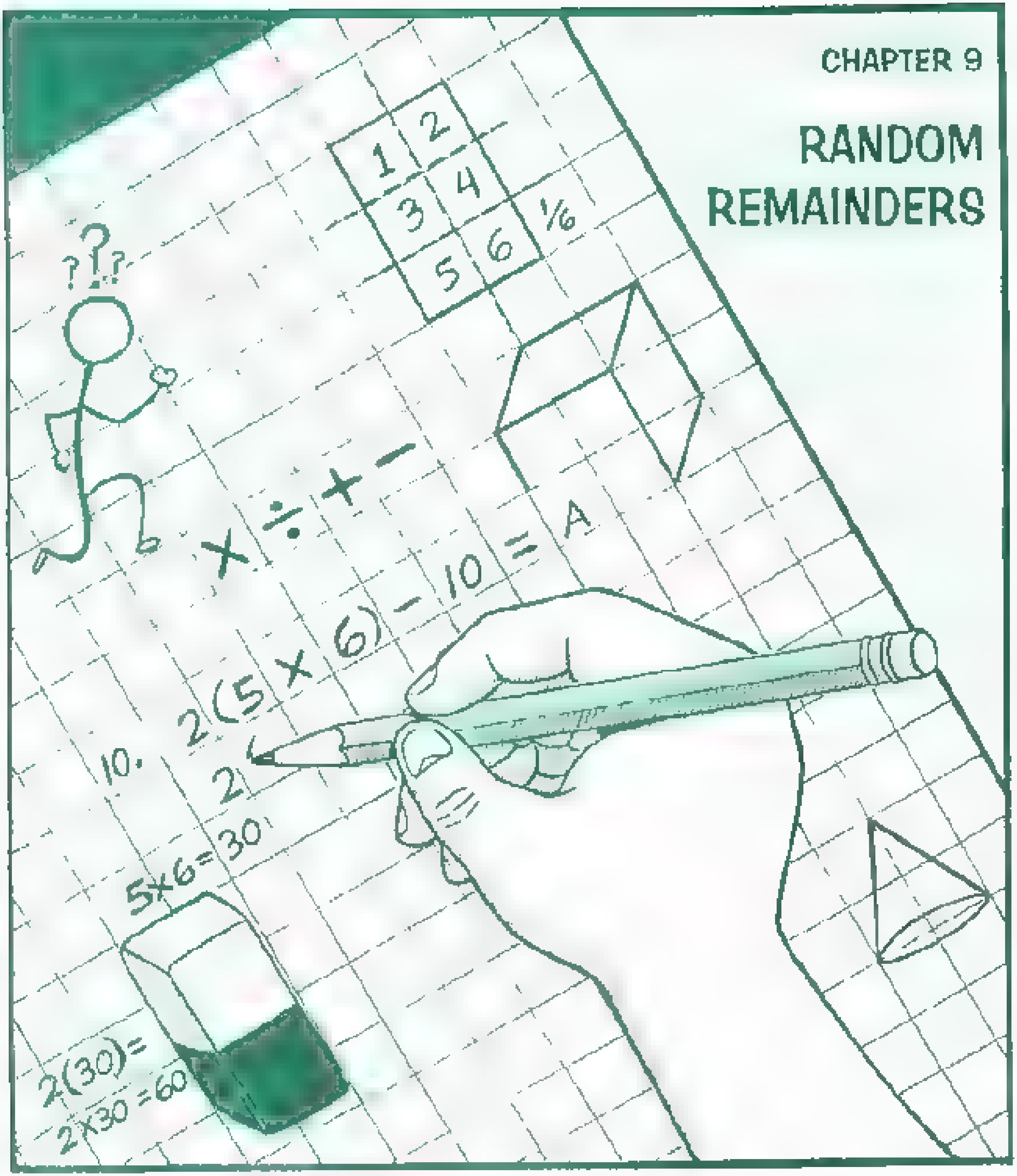
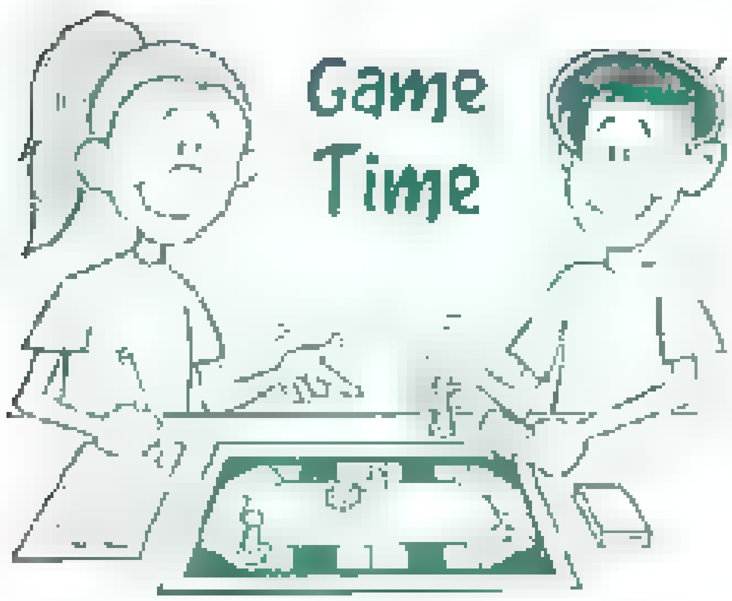
# NOTES



CHAPTER 9

# RANDOM REMAINDERS






## SO WHAT'S LEFT?

You may [illegible] remainder [illegible] It's wha[illegible] left over after a division problem. For examp[illegible] ÷ 8 = 4 with a remainder [illegible] 3. 8 goes in[illegible] 4 times [illegible] there is still 3 left over.

[illegible] this chapter are the remainder of the book [illegible] we've crammed [illegible] that you [illegible] in any of the previous chapters.

## S P E L L U L A T O R

Work out each problem [illegible] calculator. When you are through figuring, turn the calculator upside down to read the answer to the clue in parentheses. **NOTE.** Some [illegible] decimal point with two zeros. Ignore those because it may make your answer difficult to read.

9,645 / 3 = (small, medium, or large) ______________

142 x 5 = (petroleum) ______________

1,879 x 3 = (what you walk on) ______________

10,000 – 4,662 = (honey makers) ______________

50,029 – 15,023 = (barnyard animal) ______________

206 + 206 + 206 = (the opposite of tiny) ______________

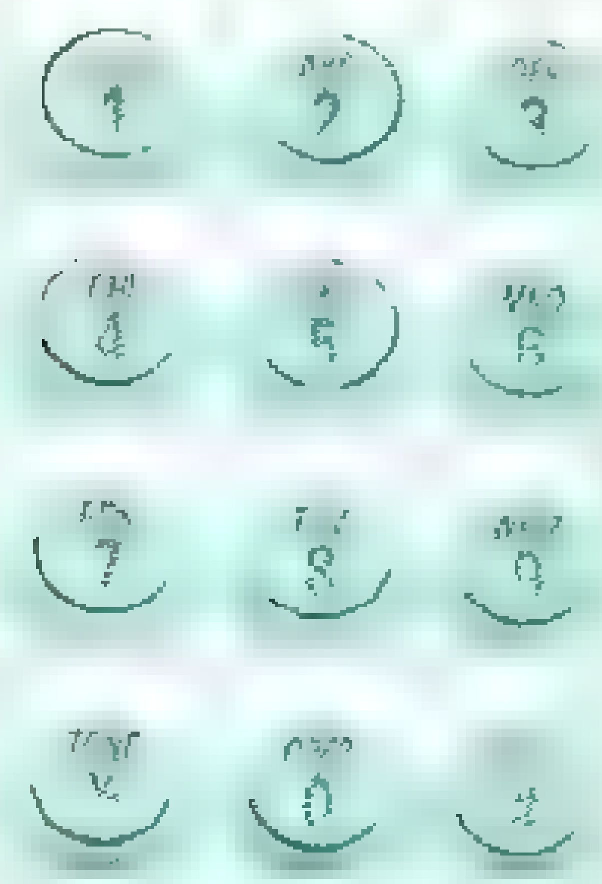
188,308 + 188,308 = (laugh in a silly way) ______________

10 + 13 = (not hard) ______________

926 x 2 x 2 = (an empty space) ______________

# Calling Code

Sam is calling his friends to invite them to a party. Using the phone keypad as a decoder, figure out the names of the people invited. Each phone button has several letters on it, so be sure to look for the number after the slash. It tells you if the letter is in the second or third space. For example, 2/2 = B. A number with no slash after it means the letter is in the first space.



1. 4/3-6-2 6/2-8/2 8 8
2. 3/3-7/2-2-6/2-5/2

   6/2 7/3-8-3/2-4/3-6/2
3. 8-3/2-3 3/2 2/2-3/2-2-7/2
4. 8-2-3/3-3/3-9/3 7-8/2-5/3-5/3
5. 3-7/2-3/2-9 2 2/2-6/3-2-8
6. 7/3-8/2-6-6-3/2-7/2 8-4/3-6-3/2
7. 7/3-2-6/2-8-2 2/3-5/3-2-8/2-7/3

## NETWORK PUZZLES

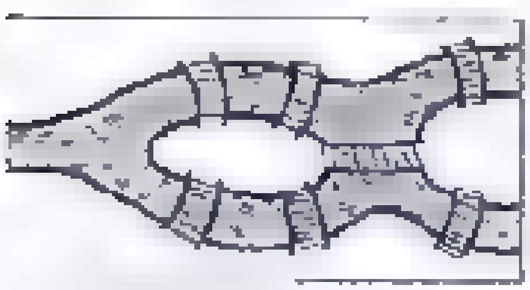
Network puzzles require you to make sense of a network that is presented to you. One of the most famous network puzzles is the **Seven Bridges of Königsberg** problem.

In Königsberg, Germany (now Kaliningrad, Russia) there was an island in the middle of a river that flowed through the city. After the river passed around the island it separated into two branches. Seven bridges were built so that the people of the city could move around.

### WORDS to KNOW

***network:*** An interconnected structure, group or system of points and lines. There are many types of networks: computer networks, railroad networks, bridge networks, internet networks, and so on.

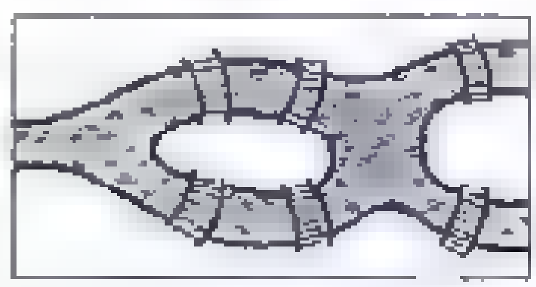
*A map of the seven bridges of Königsberg looks like this:*



**Can you walk around the city and only cross each bridge once?**

Try it! Trace the map of the city on a sheet of paper and "walk" around the city with a pencil so that you trace over each bridge once and only once without lifting your pencil.

*If the original problem seems hard, sometimes it helps to solve a simpler version:*

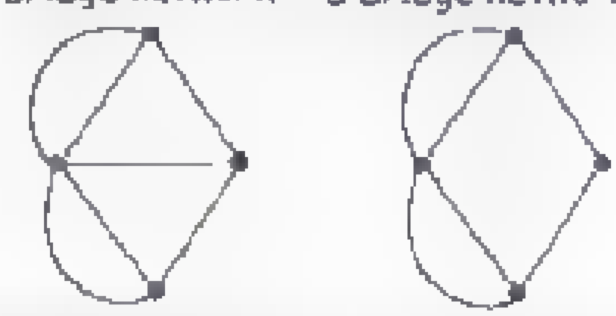


**Suppose they built six bridges in Königsberg. Could you walk around the city and only cross each bridge once?**

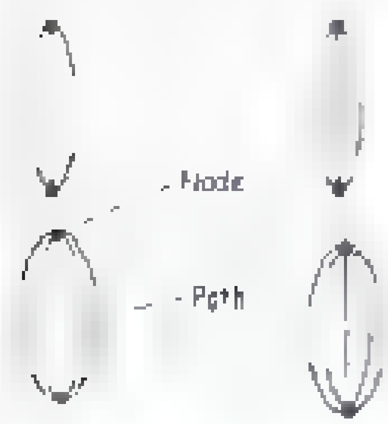
Why is this question easier? How is it different from the first question?

**Q: What's the matter with the math book?**
**A: It has problems.**

## An Approach That Makes Sense




**Here's a method to help you with any number of bridge network problems—or any network problems, for that matter.** First, turn the city map into a diagram, where each circle is a piece of land and each line is a bridge.




Each dot of land is called a node. You start and end [illegible] the place you started and travel along the lines that connect them (the paths).

**Here is the general rule for solving a network.** [illegible] and count how many paths lead out of that node. An even node has an even number of paths [illegible] odd number of paths. A network can be solved (traced without lifting a pencil and crossing each path exactly once) if the number of odd nodes is 0, 1, or 2.

## Practice Solving Networks

For each of the following network puzzles, first figure out if it can be solved by [illegible] the number of odd nodes. If it can be solved, then mark it with arrows to show how to trace around it and cross each path exactly once.

FUN FACT

### It's Perfect!

Have you ever heard of "perfect numbers"? They are whole numbers that are equal to the sum of their proper divisors. For example, 6 is a perfect number because 6 = 1 + 2 + 3. Other perfect numbers include 28, 496, 8128, 33550336, 8589869056, 137438691328, 2305843008139952128, and 2658455991569831744654692615953842176. How do you think they calculated that one?

## Number Patterns

See if you can figure out the next number in each of these eight sequences:

Sequence 1: 4, 6, 8, 10, 12, ☐

Sequence 2: 5, 10, 15, 20, 25, ☐

Sequence 3: 3, 7, 11, 15, 19, ☐

Sequence 4: 4, 9, 16, 25, 36, ☐

Sequence 5: 3, 6, 12, 24, 48, ☐

Sequence 6: 1, 3, 9, 27, 81, ☐

Sequence 7: 1, 1, 2, 3, 5, 8, 13, ☐

Sequence 8: 1, 2, 6, 24, 120, ☐

### HOPSCOTCH MATH

Bet you've never played hopscotch this way! Use the numbered hopscotch board here and for each "turn" add up the numbers in the spaces on which you hop.

• For example, on your first turn you would "hop" over number 1, so don't count it. • As you keep hopping up the board, add 2+3+4+5+6+7+8+9+10. • Turn around and hop back down the board adding 9+8+7+6+5+4+3+2 for a total of 98 points. • On your second turn you would hop on 1, hop over 2 (so don't count it), and keep on going.

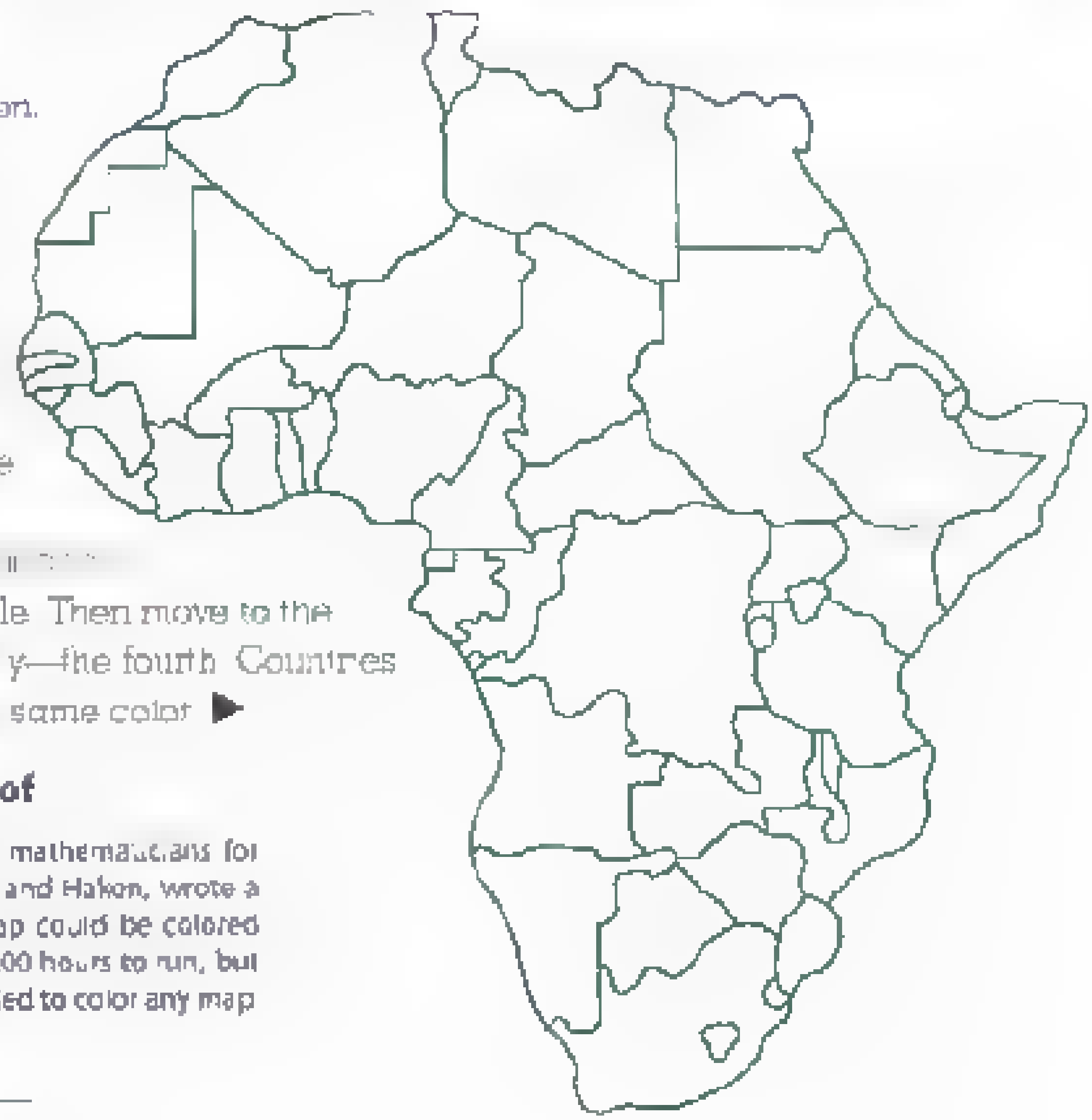


**QUESTION:**
How many turns would it take to get 380 points?

## COLOR MY WORLD

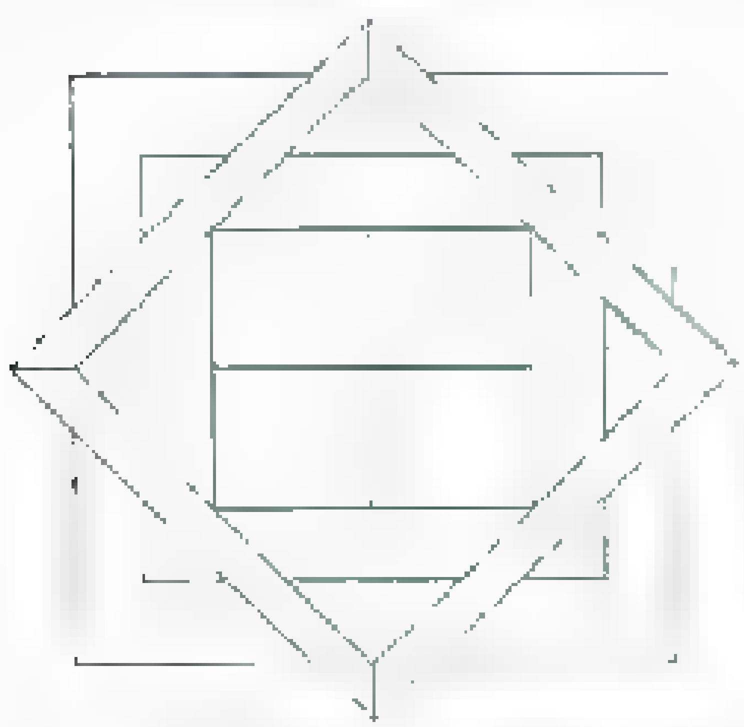
In 1893, Francis Guthrie, a student, wondered whether any map could be colored with four [illegible] to illustrate the borders: countries that share a border must be different colors.

Try to color the map of Africa using only four colors. This may be harder than it sounds, but it isn't impossible! **HINT:** Choose one [illegible] fill in as many countries as possible. Then move to the second color, the third, and—finally—the fourth. Countries that share a border cannot be the same color. ▶



### The Burden of Proof

This seemingly simple question perplexed mathematicians for years. In 1976, two mathematicians, Appel and Haken, wrote a computer program to determine if any map could be colored with four colors. The program took over 1,200 hours to run, but finally verified that only four colors are needed to color any map.



### Crazy Quilting

◀ This quilting figure isn't a map, but the same idea is true—you should be able to color all the sections using only four colors, so that no two sections that touch are the same color. **HINT:** This puzzle is easier if you start from the middle and work your way out.

## AN UNSOLVED MYSTERY

The puzzles in this book have answers, but there are some famous mathematical puzzles to which no one knows the answer yet. Maybe you will solve a famous unsolved puzzle.

One [illegible] puzzle is known as the **Goldbach's Conjecture.** The conjecture is as follows: **All even numbers greater than 2 can be written as the sum of two prime numbers.** (Remember, a prime number is one that can only be divided by 1 and itself.)

No one has proved that this is always true or disproved it by finding an example that doesn't work (a counter example). Here are examples that illustrate Goldbach's Conjecture:

**6 = 3 + 3**
**8 = 3 + 5**
**10 = 3 + 7 or 5 + 5 (there may be more than one answer)**

**Can you find two prime numbers that add up to the following even numbers? (You may want to use the prime numbers you found in Chapter 3.)**

| | |
|---|---|
| 12 = ____ + ____ | 34 = ____ + ____ |
| 14 = ____ + ____ | 46 = ____ + ____ |
| 16 = ____ + ____ | 58 = ____ + ____ |
| 20 = ____ + ____ | 60 = ____ + ____ |
| 24 = ____ + ____ | |

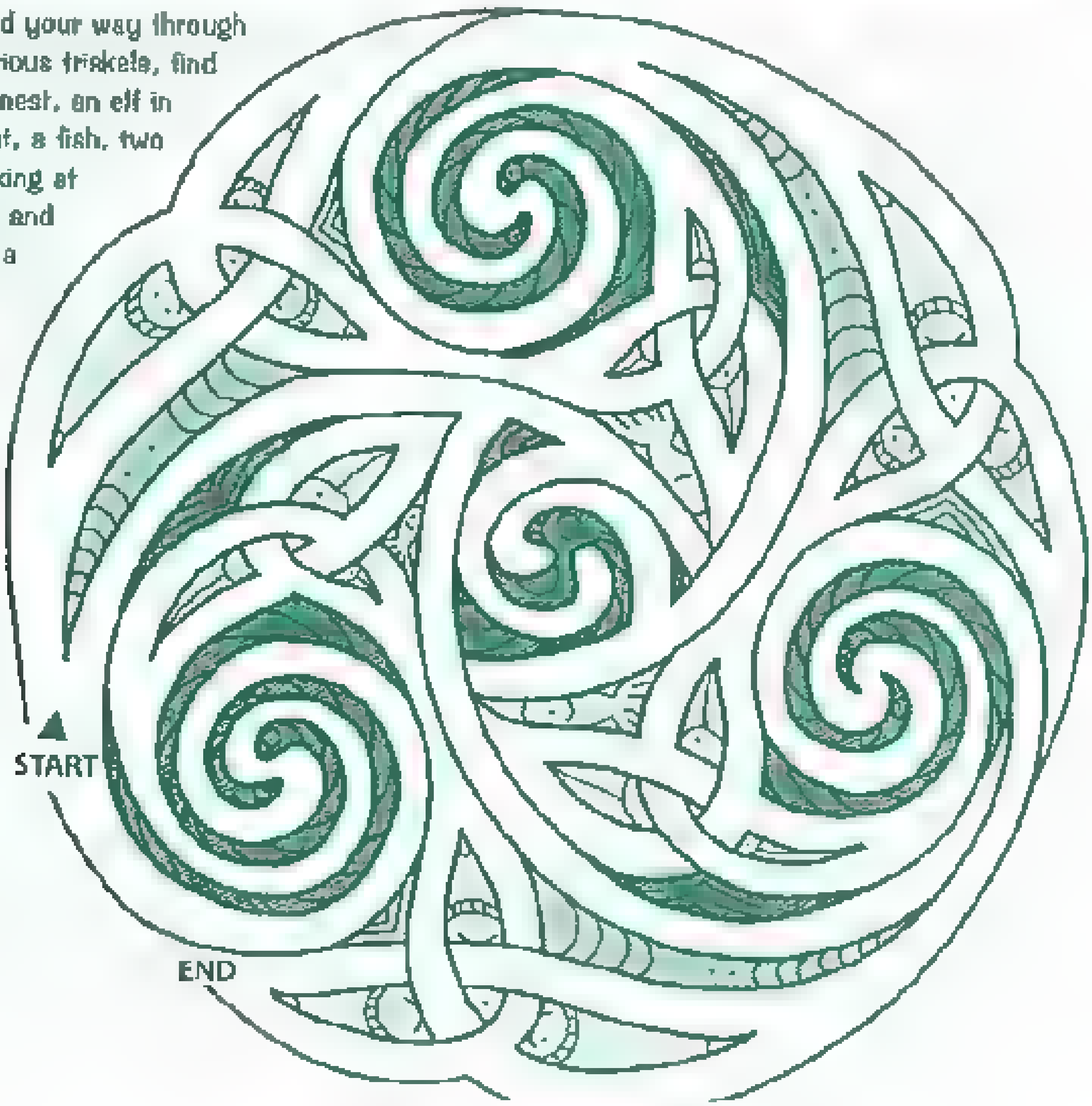
### Young Math Geniuses

Solving famous puzzles is not only for mathematicians with a lot of education. [illegible] 1995 two eighth graders found an original solution to this problem: Divide any line segment into a set number of equal parts using only a compass and straightedge. Variations of this problem date back to the time of Euclid (300 B.C.). Find out about their solution at www.glucademy.org/GLoD

# Around, and Around, and Around We Go

This triple whirling pattern, or triskele, is a very old and powerful symbol. Since the ancient Celtic people of Ireland thought that the number 3 was both sacred and magical, they may have believed that this symbol brought good luck and protection from evil. Sometimes the artists would add tiny lines or dots to make the pattern look like an animal, a flower, or a human face.

**As you wind your way through this mysterious triskele, find a bird in a nest, an elf in a pointy hat, a fish, two snakes looking at each other, and a bird with a big beak.**

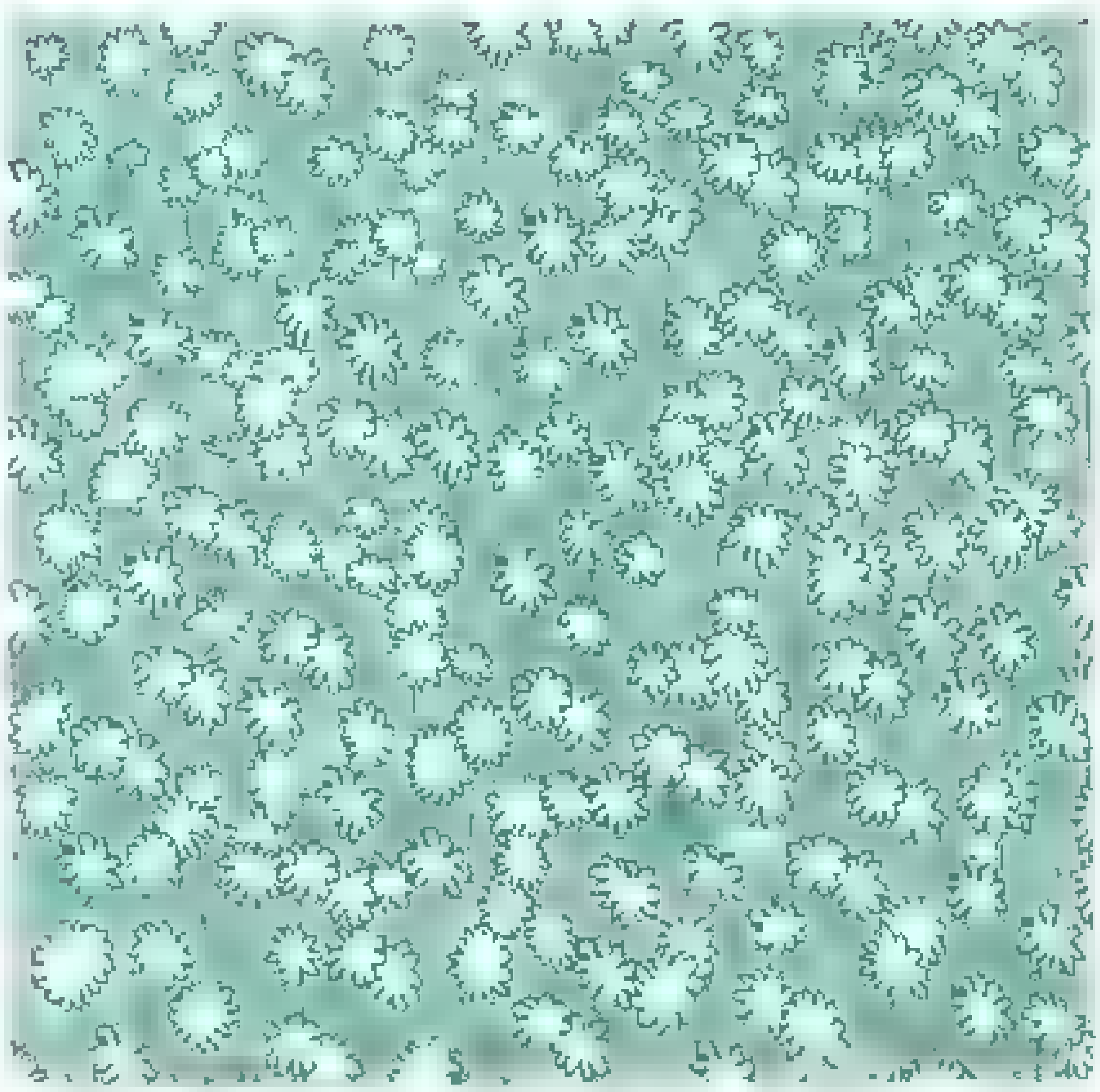


# That's Just about Right

Take a wild guess at how many daisies are in this field. Write your wild guess here ________

Here's a handy [illegible] help you [illegible] correct number of dai[illegible].

1. [illegible] boxes. The boxes don't have to be perfectly square, and it doesn't matter how many [illegible]
2. [illegible]
3. [illegible] the total number of boxes on the picture



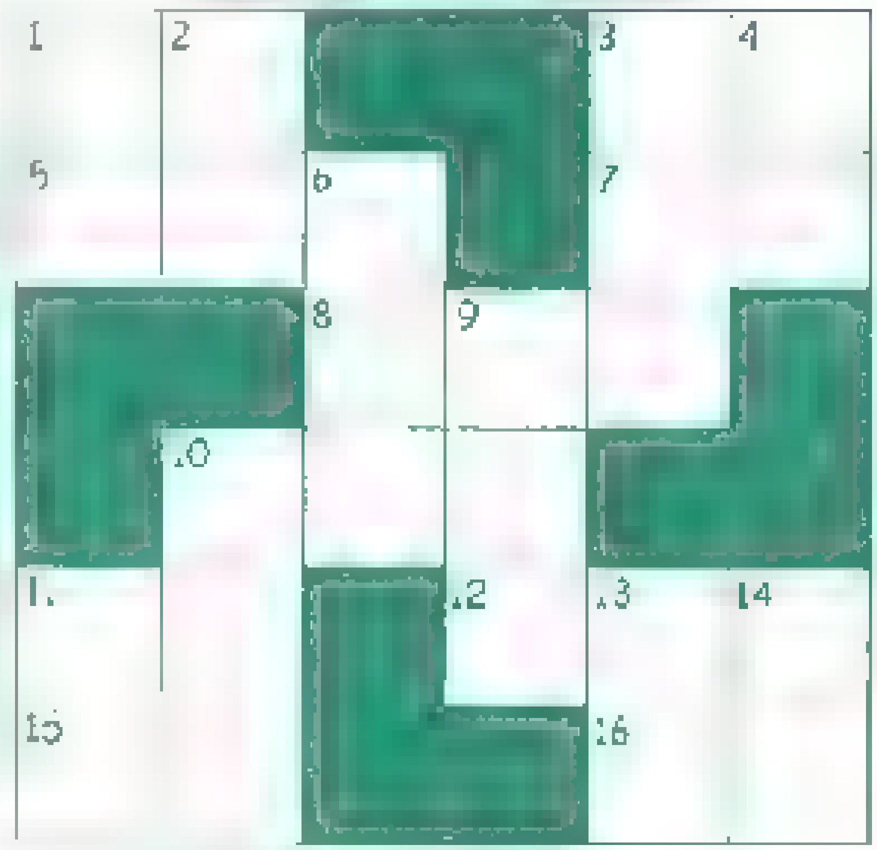
This answer is based on reasoning and is called an estimate. An estimate is still a guess, but it's an educated guess, not just a wild one! Write your estimate of the number of flowers here: ________

## How do your two answers compare?

[illegible]

# The Very Last Cross-Number Puzzle

Well, you've come to the end of the book.
But before you go on your way, here is one last cross-number [illegible]



**Across**

1. If "a cat has [illegible] lives do nine cats h[illegible]?
3. How many cards are in a [illegible] deck?
5. The number that is two numbers [illegible], 5, three times in a row.
7. How much is seven sevens?
8. How many days are in a ye[illegible]?
10. How many Dalmatians are in that classic movie?
11. How many hours are in a day?
12. What's between 5 and 8?
15. It takes eight rows of eight checkers to fill a checkerboard. How many checkers is that?
16. How many states are in the United States?

**Down**

1. A [illegible] years. How many years [illegible]
2. XVII in Arabic numbers.
3. How would you see "quarter to six" on a digital clock?
4. A [illegible] etc.
6. What's an early bedtime?
9. Th[illegible]
[illegible] a gross. If you have a gross of pencils, how many pencils do you have?
1[illegible]. Take an unlucky number and double it.
13. [illegible] equal three [illegible]?
1[illegible]. [illegible] and two nickels?

# NOTES

APPENDIX A

# Glossary of Math Terms

### Arabic numerals

A numeral system that relies on ten digits—0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The Arabic numerals are what we use today.

### Averages

There are three ways to calculate the average. The **mean** is what you get if you add up all the values and divide the total by the number of values. This is typically what people mean when they use the word **average**. The **median** of a set of values is the value in the middle of the set when the set is written in order. The **mode** of a set of values is the value that occurs most often. Mode is usually used as the average when the values are not numbers, like eye or hair color.

### Binary system

A numeral system that uses 1 and 0, where 1 represents "on" and 0 represents "off." Our computers rely on the binary system.

### Calculate

This term originated from the Greek word *kalyx*—pebble or small stone—because a long time ago, Greeks used small stones to do simple calculations.

### Cent–

100

### Deci–

### Deka–

[illegible]

### Digit

A number, but also a word for "finger." Coincidence? We think not. It's very likely that the first people to start counting used their fingers—just as little kids continue to do today.

**Fibonacci series:** A series of numbers that begin with 1, 1, 2, 3, 5, 8, 13, and so on, where the two first numbers add up to the third (1+1=2), the second and third numbers add up to the fourth (1+2=3), and so on. This sequence was devised by Leonardo Pisano (Fibonacci), a mathematician who lived in Pisa, Italy between 1170 and 1250.

### Geometry

A study of physical shapes. Literally, **geometry** [illegible] measure the Earth.

### Hecto–

[illegible]

### Irrational

In mathematics, irrational numbers are those that cannot be represented as a fraction; they go on forever and ever with no repeating pattern.

### Kilo–

1,000.

### Mathematics

Based on the Greek word *manthanein* (to learn), mathematics measures and describes the world with numbers and symbols.

### Milli–

[illegible]

### Network

An interconnected structure, group, or system of points and lines. There are many types of networks: computer networks, railroad networks, bridge networks, Internet networks, and so on.

### Obelus

The symbol "÷" used to indicate division. The word comes from the Greek word *obelos*, for spit or spike, a pointed stick used for cooking. The symbol has been used as a division symbol since around 1650.

### Pi (p)

This number represents the ratio of circumference (the distance around the circle) and diameter (the distance across the circle). This number is the same for all circles, no matter what their size.

### Polygon

A geometrical figure with three or more sides. The term **polygon** is from the Greek roots *poly* (many) and *gonus* (knees). Greeks thought of a polygon as a shape of many angles, which look like bent knees.

**Prime number**

A number that is divisible by just two different numbers, 1 and itself. Seven is prime because it can only be divided by 1 and 7. Eight is not a prime number because besides 1 and 8, it is also divisible by 2 and 4. The word **prime** comes from primus, the Latin word [illegible]

**Probability**

The chances of something happening; actually, the ratio of the number of times it could happen compared to the number of possible things that could happen. For example, when you throw dice, you have 1/6 chances of get-[illegible] particular number.

**Roman numerals**

Numerals I (1), V (5), X (10), L (50), C (100), D (500), M (1,000), and so on, which were invented by the Romans and are still used today on clocks and in a few other cases.

**Solar system**

The sun, nine planets that revolve around it—Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune, and Pluto—as well as all of their respective moons, comets, and asteroids.

**Solidus**

The slanted bar "/" used for fractions and division. During the Roman Empire, the solidus was a gold coin. On the reverse of the coin was a picture of a spear bearer with the spear going from lower left to upper right. This spear became the symbol for fractions and division.

**Tallies**

Simple lines that each represent one object and are used for simple counting.

APPENDIX B

# Read All about It!

### Anno's Mysterious Multiplying Jar

by Masaichiro and Mitsumasa Anno (Philomel Books, 1983)

### Counting on Frank

by Rod Clement (Gareth Stevens Publishing 1991) explores how you and your imagination can make counting [illegible]

### Easy Origami

by John Montroll (Dover Publications, 1992) shows you how to make basic origami folds and how to make fish, birds, boats, and [illegible]

### Grandfather Tang's Story, A Tale Told with Tangrams

by Ann Tompert (Crown Publishers, 1990) Changing tangram pictures tell the story about two foxes and a dangerous game that might get one of them killed

### The Greedy Triangle

by Marilyn Burns (Scholastic, 1994) What [illegible] the triangle wanted more sides? In this story, a triangle changes into a quadrilateral, then a pentagon, then a hexagon, and keeps growing. Will it ever be happy? [illegible] out by reading *The Greedy Triangle*.

### How Much Is a Million?

by David M. Schwartz (Scholastic, 1985) shows a million, a billion, and a trillion in pictures of children, goldfish, and stars. If you can imagine a million, then how much bigger is a billion?

### If You Made a Million

by David M. Schwartz (Scholastic, 1989) Read this book if you are curious to explore what a million dollars looks like and what it [illegible]

### Is a Blue Whale the Biggest Thing There Is?

by Robert E. Wells (Albert Whitman & Company, [illegible]) [illegible] mind [illegible] really big [illegible]

### Jumanji

by Chris Van Allsburg (Houghton Mifflin [illegible] 1981). What if a roll of dice and a move [illegible] the game board was real? Read and figure out the probability of the next move [illegible] made into a movie—did you see [illegible]

### The King's Chessboard

by David Birch (Puffin Pied Piper, 1988). When [illegible] the king to double his gift at [illegible] day [illegible] king really grant the wise [illegible] the power of [illegible] king that the humble [illegible] has a lesson worthy even of kings

### Math Curse

by Jon Scieszka and Lane Smith (Viking, 1995). [illegible] math problem [illegible] didn't like math. In this funny story, the clever girl eventually breaks the math curse

### Origami, Plain and Simple

by Robert Neale and Thomas Hull (St. Martin's Press, 1994) is a great book for beginners. It even shows [illegible] an entire chess set from paper.

### Roman Numerals I to MM

by Arthur Geisert (Houghton Mifflin Company, 1996)

### Sir Cumference and the First Round Table

by Cindy Neuschwander (Charlesbridge Publishing, 1997). In this fun tale, King Arthur's knights try out a variety of shapes for tables until a knight's son finds the best solution.

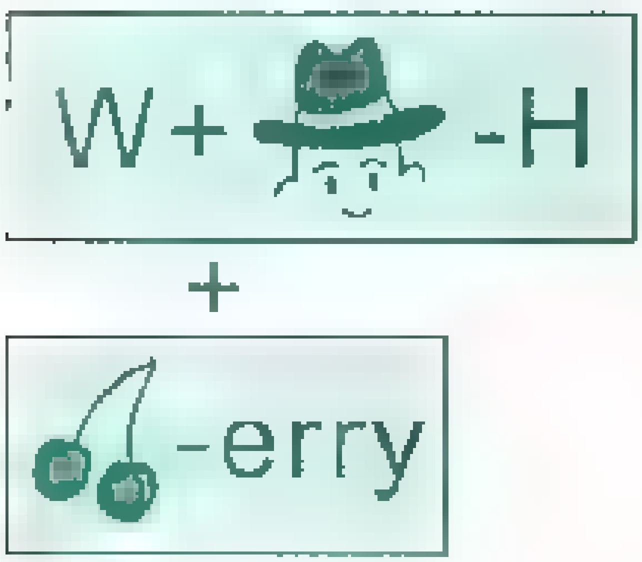
### What's Smaller Than a Pygmy Shrew?

by Robert E. Wells (Albert Whitman & Company, 1995) can help you imagine how [illegible] really small—down to a particle that is smaller than a molecule

# PUZZLE ANSWERS

page 2 • **Roman Numerals**


Answer: CLOCK




Answer: WATCH

page 4 • **When in Rome**

| | | | | | | |
|---|---|---|---|---|---|---|
| 149 | 14 | 2660 | 922 | 1606 | 14 | 61 |
| N | A | T | U | R | A | L |
| 149 | 922 | 411 | 29 | 8 | 1606 | 751 |
| N | U | M | B | E | R | S |

page 5 • **Hidden Numbers**

1. I love my computer—when it works!
2. Beth reeked of smoke after sitting by the campfire.
3. My mother likes to weigh tomatoes on every scale in the store.
4. Annie was even early for school last week.
5. We can stuff our dirty backpacks in your tent.
6. We like the mirrored maze room at the fun park.

page 6 • **Practice Your Digits**

| | | | | |
|---|---|---|---|---|
| 3 | 3 | 4 | 2 | 3 |
| 4 | 1 | 1 | 2 | 2 |
| 2 | 4 | 3 | 1 | 4 |
| 2 | 3 | 1 | 4 | 3 |
| 4 | 2 | 1 | 2 | E |

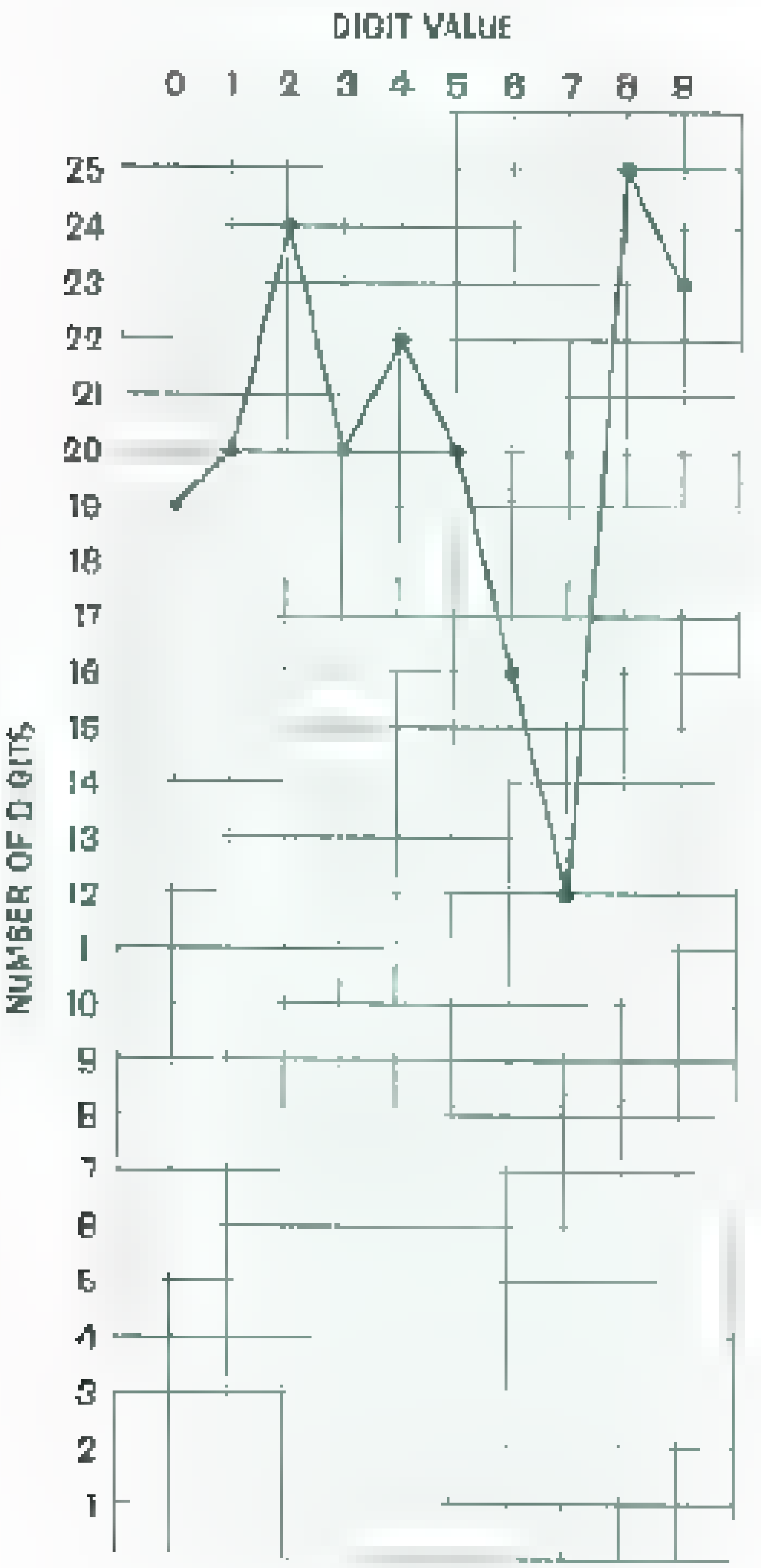
| | | | | | |
|---|---|---|---|---|---|
| 4 | 2 | 1 | 2 | 3 | 4 |
| 3 | 3 | 4 | 2 | 3 | 4 |
| 4 | 2 | 4 | 2 | 4 | 3 |
| 3 | 2 | 1 | 3 | 2 | 4 |
| 2 | 2 | 4 | 2 | 3 | 4 |
| 4 | 1 | 4 | 1 | 2 | E |

page 7 • **On or Off?**

T O O T H  T H  R T Y

# PUZZLE ANSWERS

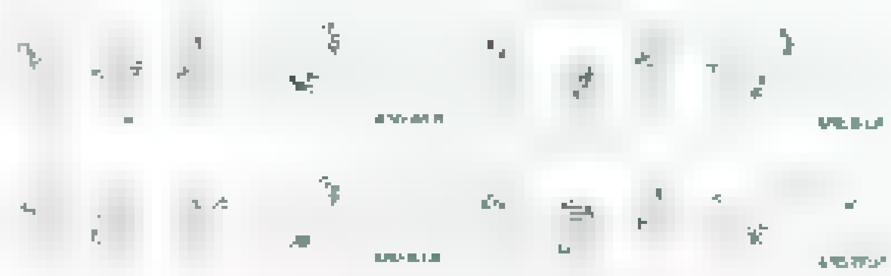
**pages 10–11 • Making Sense of the Irrational**




**page 12 • Let's Get Packing**

1. Call Kelly Short for directions to State park
2. Buy sunblock and bug spray
3. Charge the flashlight batteries
4. Fill water bottles, make snacks, and get chocolate
5. Pack ponchos and extra socks
6. Find binoculars and bird books

**page 13 • See What I Mean?**



**page 16 • Clock Math**

| | |
|---|---|
| 10 + 12 = T | 11 + 4 = H |
| 3 + 4 = O | 8 + 3 = U |
| 10 + 9 = O | 11 + 3 = R |
| 1 + 3 = T | 11 + 1 = T |
| 5 + 2 = H | 12 + 12 = Y |

# PUZZLE ANSWERS

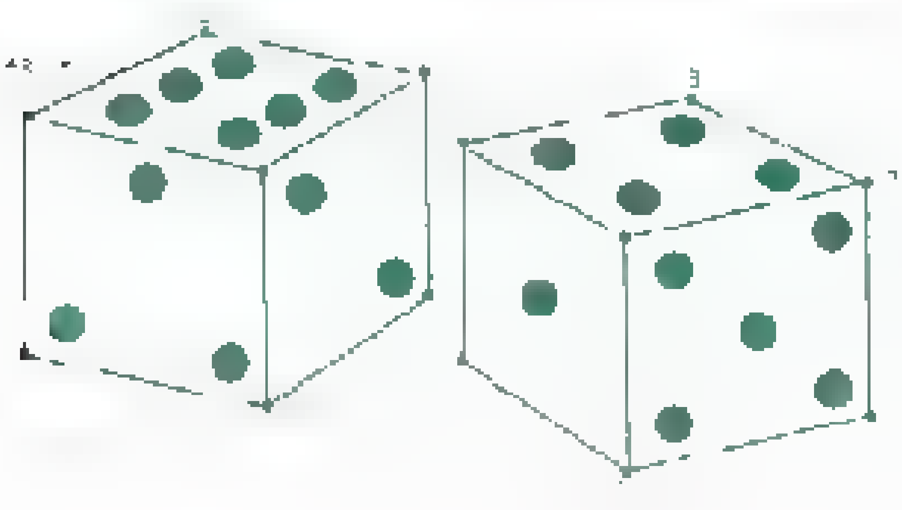
**page 18 • Numbers with Direction**

| 14 | 21 | 13 | 2 | 5 | 18 |
|---|---|---|---|---|---|
| N | U | M | B | E | R |

| 12 | 9 | 14 | 5 |
|---|---|---|---|
| L | I | N | E |

**page 20 • Connect the Dots**

| Do in order | Addition | Subtraction |
|---|---|---|
| A | 3 + 5 = 8 | 3 -5 = 2 |
| B | 4 + 2 = 7 | 4 -8 = 2 |
| C | -1 + 5 = 4 | 5 2 = 3 |
| D | 4 + 8 = -4 | 4 - -2 = -2 |
| E | 7 + 2 = 5 | 7 - -2 = 9 |
| F | -3 + 3 = 0 | -2 - -2 = 0 |
| G | 3 + 4 = 7 | -3 - 4 = -7 |
| H | -2 + -2 = 4 | -1 -1 = -2 |



**page 21 • Magic Squares**

There is more than one way to complete the magic squares, but the solutions will be similar. Here is one way of solving each magic square.

**Magic Square 5**

| | | |
|---|---|---|
| 8 | 3 | 4 |
| 1 | 5 | 9 |
| 6 | 7 | 2 |

**Magic Square 10**

| | | |
|---|---|---|
| 16 | 6 | 8 |
| 2 | 10 | 18 |
| 12 | 14 | 4 |

**Magic Square 9**

| | | |
|---|---|---|
| 15 | 5 | 7 |
| 1 | 9 | 17 |
| 11 | 13 | 3 |

**Magic Square 0**

| | | |
|---|---|---|
| 3 | -2 | -1 |
| -4 | 0 | 4 |
| 1 | 2 | -3 |

**Magic Square 1**

| | | |
|---|---|---|
| 4 | -1 | 0 |
| -3 | 1 | 5 |
| 2 | 3 | -2 |

**Magic Square 4X4**

| | | | |
|---|---|---|---|
| 16 | 2 | 3 | 13 |
| 5 | 11 | 10 | 8 |
| 9 | 7 | 6 | 12 |
| 4 | 14 | 15 | 1 |

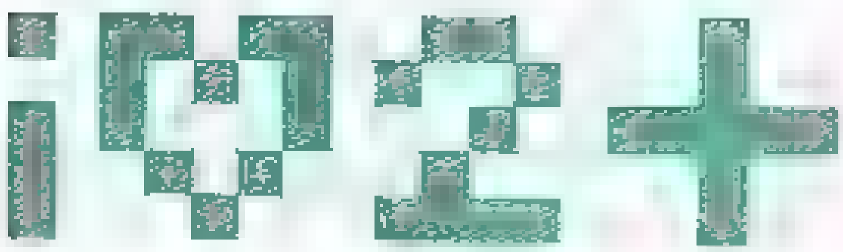
**pages 24–27 • Solve a Cross-Number Puzzle**

# PUZZLE ANSWERS

**page 28 • Cross Sums**

| | 9 | 24 | | 27 | 13 |
|---|---|---|---|---|---|
| 6 | 4 | 1 | 12 | 5 | 7 |
| 11 | 5 | 6 | 15 / 6 | 9 | 6 |
| 17 / 5 | | 3 | 6 | 8 | 14 |
| 16 | 7 | 9 | 12 | 4 | 8 |
| 13 | 8 | 5 | 4 | 1 | 3 |

**page 29 • It's My Favorite**



I LOVE TO ADD

**page 35 • Multiplication Boxes**

| | | |
|---|---|---|
| -4 | 5 | [illegible] |
| -9 | [illegible] | [illegible] |
| | 30 | |

| | | |
|---|---|---|
| 7 | 10 | |
| | | -11 |
| | 10 | |

| | | |
|---|---|---|
| | | 21 |
| -5 | | 10 |
| | -6 | |

| | | |
|---|---|---|
| -3 | | 18 |
| | | |
| -21 | | 1008 |

**page 36 • Musical Math**

$5\overline{)95}$ = 19 $4\overline{)72}$ = 18 $3\overline{)87}$ = 29 $6\overline{)84}$ = 14

$7\overline{)175}$ = 25 $3\overline{)141}$ = 47

$8\overline{)168}$ = 2 $6\overline{)264}$ = 44

$8\overline{)264}$ = 33 $6\overline{)186}$ = 3

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Y 31 | A 45 | U 47 | O 16 | R 19 | N 12 | Y 11 | M 29 |
| R 14 | E 13 | B 15 | R 33 | M 25 | G 16 | A 22 | W 14 |

What comes before a tuba?
Answer: A one-ba!

**page 41 • The Sieve of Eratosthenes**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

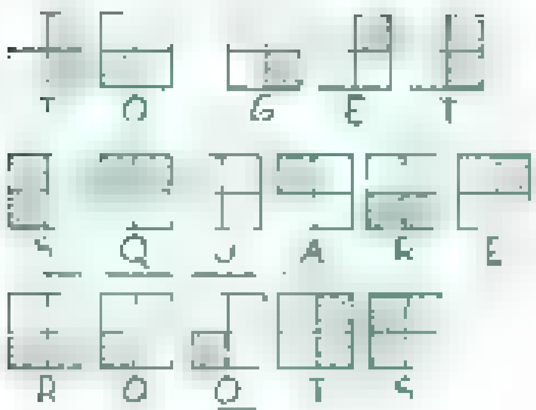
Prime Numbers: 2 3 5, 7 11 13, 17 19 23 29 31 37
41, 43 47 53 59 61, 67 71, 73 79, 83 89 97

# PUZZLE ANSWERS

**page 45 • Squares and Radicals**

$5^2 = 25$, $6^2 = 36$, $7^2 = 49$, $8^2 = 64$, $9^2 = 81$, $10^2 = 100$, $11^2 = 121$, $12^2 = 144$, $13^2 = 169$, $14^2 = 196$, $15^2 = 225$, $16^2 = 256$, $17^2 = 289$, $18^2 = 324$, $19^2 = 361$, $20^2 = 400$

**page 46 • Goofy Gardener**




**page 47 • A Radical Sign**

$\sqrt{9} = 3$; $\sqrt{16} = 4$; $\sqrt{25} = 5$; $\sqrt{49} = 7$; $\sqrt{64} = 8$; $\sqrt{81} = 9$; $\sqrt{100} = 10$

**page 48 • On Your Mark!**

$\boxed{\sqrt{9}}$ 3 to get ready,
$\boxed{-3+4}$ 1 for the money,
$\boxed{2^2}$ 4 to go
$\boxed{4 \div 2}$ 2 for the show,

1 for the money,
2 for the show,
3 to get ready,
4 to go!

**page 50 • How Four Can You Go?**

$2 = 4 \div 4 + 4 \div 4$
$3 = (4 + 4 + 4) \div 4$
$4 = (4 - 4) \times 4 + 4$
$5 = (4 \times 4 + 4) \div 4$
$6 = (4 + 4 + 4) \div \sqrt{4}$
$7 = 4 + [4 - (4 \div 4)]$
$8 = 4 + 4 + 4 - 4$
$9 = 4 + 4 + (4 \div 4)$
$10 = 4 \times 4 - 4 - \sqrt{4}$
$11 = 44 \div (\sqrt{4} + \sqrt{4})$
$12 = \sqrt{(4 \times 4 \times 4)} + 4$
$13 = (44 \div 4) + \sqrt{4}$
$14 = 4 + 4 + 4 + \sqrt{4}$
$15 = (4 \times 4) - (4 \div 4)$

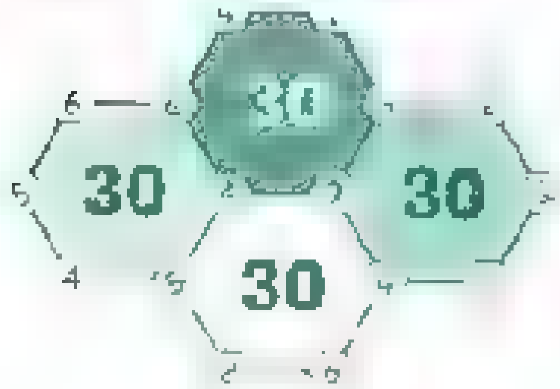
**page 51 • Your Number's Up**

1 ONE STEP AT A TIME / HOLE IN ONE / A LONELY NUMBER
2 TEA FOR TWO / DICE IN A MONOPOLY GAME
3 LITTLE PIGS / THREE MUSKETEERS
4 QUARTERS IN A DOLLAR / 4 PRIVET DRIVE (The Dursleys' address in *Harry Potter*)
5 POINTS ON A STAR / OLYMPIC RINGS / SCHOOL DAYS IN A WEEK
6 HALF A DOZEN / LEGS ON AN INSECT
7 DWARVES WITH SNOW WHITE / DAYS IN A WEEK
8 ARMS ON AN OCTOPUS / OUNCES IN A CUP
FIGURE 8 IN ICE SKATING / EIGHT SIDES ON A STOP SIGN
9 CAT LIVES / PLANETS IN OUR SOLAR SYSTEM
10 FINGERS OR TOES / COMMANDMENTS
11 PLAYERS ON A FOOTBALL OR SOCCER TEAM
APOLLO 11 MISSION TO THE MOON
12 MONTHS IN A YEAR / NUMBERS ON A CLOCK FACE
13 AN UNLUCKY NUMBER / STRIPES ON THE U.S. FLAG
14 FEBRUARY 14 IS VALENTINE'S DAY
15 MINUTES IN A QUARTER OF AN HOUR
16 OUNCES IN A POUND
17 TITLE OF A TEEN MAGAZINE / AMERICAN REVOLUTION STARTED IN 1776
18 HOLES ON A GOLF COURSE / AN AGE WHEN YOU ARE OLD ENOUGH TO VOTE
19 *1984* (title of a book) / BUILDING 19 (a discount store)
20 FINGERS AND TOES (all together) / *20 THOUSAND LEAGUES UNDER THE SEA* (title of a book)
21 ANOTHER NAME FOR THE GAME OF BLACKJACK
THE CENTURY WE ARE IN
22 FEBRUARY 22 IS GEORGE WASHINGTON'S BIRTHDAY
23 MICHAEL JORDAN'S NUMBER / CHROMOSOMES IN A HUMAN BEING
24 HOURS IN A DAY / TWO DOZEN
25 DECEMBER 25 IS CHRISTMAS DAY / A SILVER ANNIVERSARY
50 U.S. STATES / STARS ON THE U.S. FLAG
100 "ONE HUNDRED BOTTLES OF BEER ON THE WALL" SONG / PERFECT SCORE ON A TEST

# PUZZLE ANSWERS

**page 54 • Shape Changers**


**page 55 • Six-Sided Math**



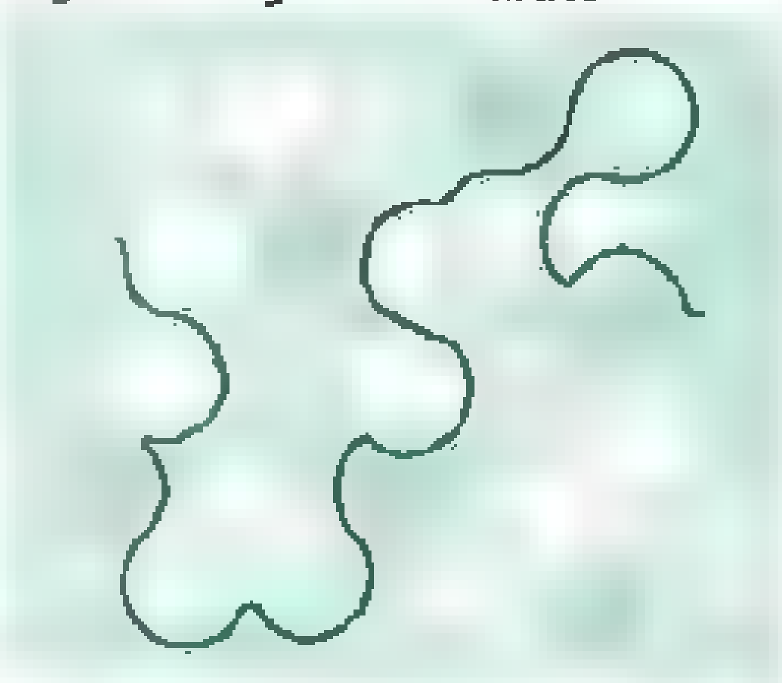


**page 56 • Hide in Plain Sight**

**page 57 • Life of the Party**

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| P | O | I | N | T | L | E | S | S |

**page 58 • Going Around in Circles**

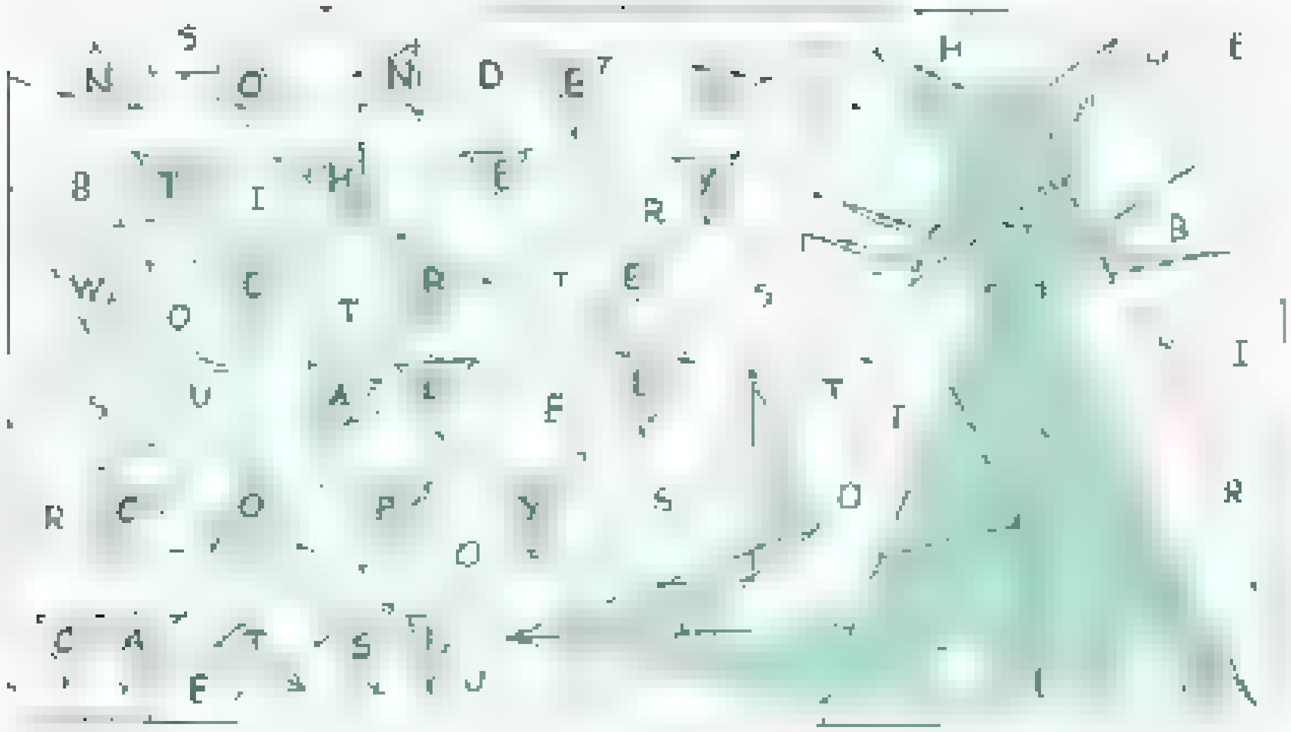


**page 61 • The Last Straw**

Tape the tips of three straws together to form a triangle that lies flat on the table. Now, take the remaining three straws and tape them so that one stands upright from each of the three corners. Finally, lean the three straws toward the center and tape all three tips together so that you have an open pyramid. This is called a "tetrahedron," and it is made up of one triangle lying flat on the table plus three more triangles for the upright side.

# PUZZLE ANSWERS

**page 62 • Get to the Point!**



Answer: None — they were all copy cats!

**pages 64–65 • On a Treasure Hunt**

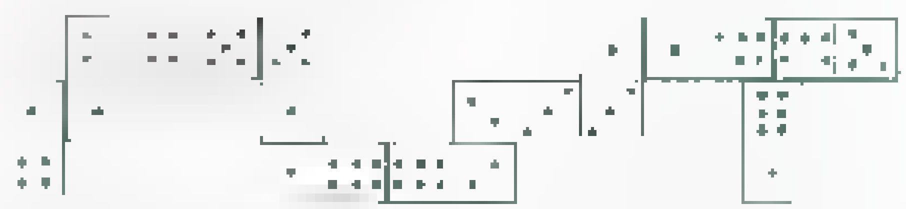
**The Windy Hill**

1. *Her father was afraid of ghosts and wouldn't want his money close to the cemetery.* Cross out the cemetery.
2. *Cash Steele did not like small dark places either.* Cross out mine shaft, well, and cave.
3. *If he buried the money, it would be to the north of his cabin.* There are no underground hiding places left north of the cabin, so he must not have buried his money. Cross out dino bones.
4. *If he hid it aboveground, it would be south of the cabin.* Both the barn and the Lone Pine are north of the cabin, so cross those out.
5. *If he didn't hide it in the well, then he didn't hide it in the windmill, either.* He didn't hide it in the well (you've already crossed it out), so you can cross out the windmill as well. Now, the only other hiding place left is Windy Hill, which is the answer.

**page 66 • A Square Deal**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 2 | 4 | [illegible] | 1 | [illegible] | [illegible] | [illegible] | 4 |
| 2 | 1 | 2 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 1 | 2 | 4 | 3 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 3 | 4 | 3 | 2 | 1 | 2 | 3 | 4 | 1 |
| 4 | 3 | 2 | 1 | **4** | 1 | [illegible] | [illegible] | [illegible] |
| 3 | 4 | 3 | 2 | 1 | [illegible] | [illegible] | 4 | 1 |
| 1 | [illegible] | 4 | 3 | 2 | 3 | [illegible] | [illegible] | 3 |
| 2 | [illegible] | 2 | 4 | 1 | 4 | [illegible] | [illegible] | [illegible] |
| 4 | 2 | 4 | [illegible] | 3 | 1 | [illegible] | 2 | 4 |

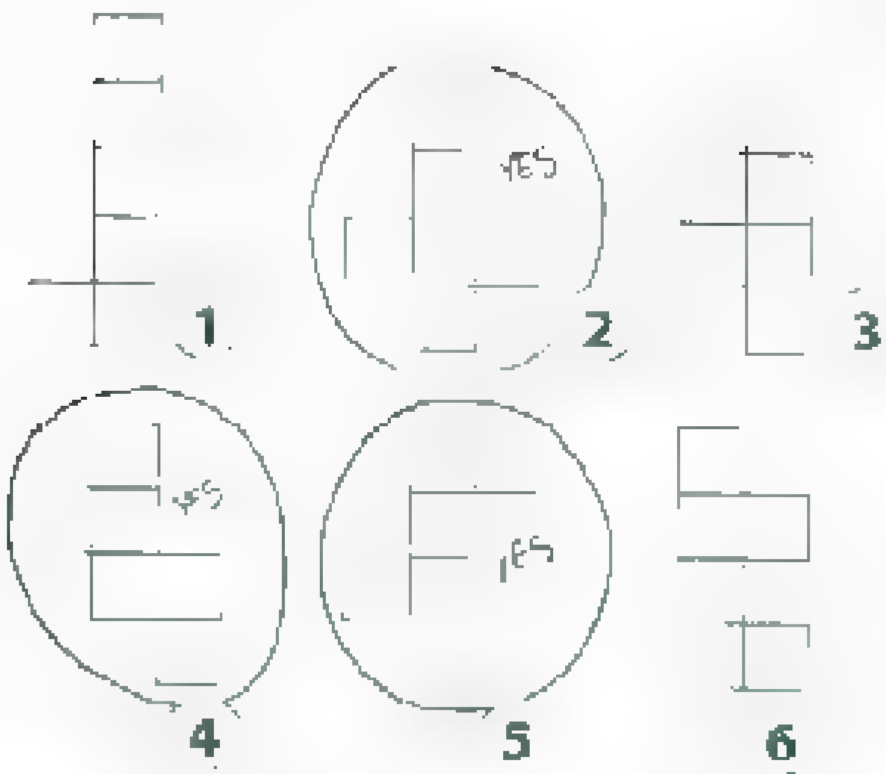
**page 66 • The Domino Effect**



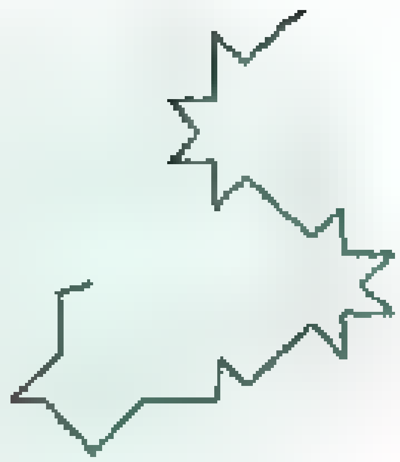
Dominoes work by matching the open numbers, so that a 1 goes next to a 1, a 2 next to a 2, and so on.

# PUZZLE ANSWERS

**page 67 • Picture This**




**page 75 • Star Power**



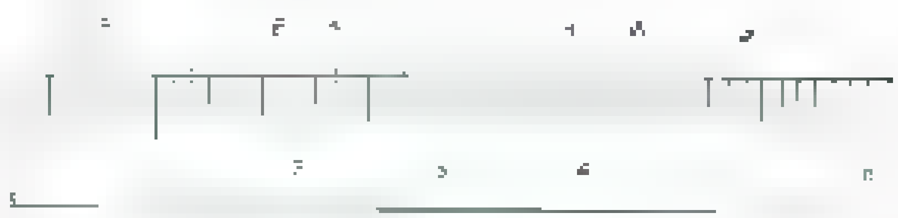
**pages 72–73 • Not by a Long Shot**
Giraffe

**page 74 • Hink Pinks**

| | |
|---|---|
| 1. Pile of games that weighs 2,000 lbs. | FUN TON |
| 2. 5,280-foot grin | MILE SMILE |
| 3. Two pastries that are each 12 inches long | SWEET FEET |
| 4. Very difficult 3 feet | HARD YARD |
| 5. Rubber ball that weighs 16 ounces | ROUND POUND |
| 6. 28.4 grams that jumps quickly | OUNCE POUNCE |
| 7. Urgent message that weighs 2.2 pounds | KILOGRAM TELEGRAM |

**pages 76–77 • A Weighty Matter**
Blue whale

**page 78 • Measuring Your ZZZs**



T O   S E E   H O W   L O N G
3, 125   2.25 1 5   3, 125 4,   2.75 1, 5

S H E   S L E P T
2, 3, 1,   2, 2 1 3

**page 79 • Lost Billions**

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| B | I | L | L | I | O | N | N | S | B | B | I |
| B | I | L | S | B | S | N | O | I | L | I | B |
| I | B | S | S | N | B | I | L | L | O | N | S |
| L | S | N | N | B | O | B | I | L | L | N | O |
| L | N | I | O | L | I | I | B | B | O | B | I |
| L | O | O | I | L | B | O | L | I | B | I | L |
| I | I | I | L | O | B | S | I | I | O | B | I |
| O | L | L | L | O | N | I | B | O | I | S | L |
| N | L | I | B | I | L | I | O | B | B | I | I |
| S | I | B | L | I | B | O | B | I | I | I | B |
| I | B | L | B | B | I | L | O | O | N | S | I |
| O | B | B | I | L | I | O | O | N | S | O | L |

# PUZZLE ANSWERS

page 80 • **Googols of Fun**

1. 7•15•15•7•15•12 8•1•19
A G O O G O L H A S

15•14•5 8•21•14•4•18•5•4
O N E H U N D R E D

26•5•18•15•19
Z E R O S

page 81 • **On Your Toes**

SHE ADDS TWO FEET

pages 84–85 • **Logical Math**

1. The *other one* is a nickel!
2. 99 + (9 ÷ 9) = 100
3. 30 ÷ 15 = 2 and 2 + 10 = 12. Note: 30 *divided by* half is not the same as 30 *divided in* half.
4. You have the three apples you took away.
5. "A pie in the face."
6. "One in a million."
7. "Six of one, half a dozen of another."
8. He weighs meat!

page 86 • **Moving Around**

Gary is moving to Boston, Harry is moving to Dallas, Larry is moving to Chicago, Mary is moving to Atlanta.

page 87 • **Taming Dragons**

| | Day 1 | Day 2 | Day 3 | Day 4 |
|---|---|---|---|---|
| **Ruby** | Smokey | Spitfire | Forktail | Blaze |
| **Jade** | Forktail | Smokey | Blaze | Spitfire |
| **Sapphire** | Blaze | Forktail | Spitfire | Smokey |
| **Topaz** | Spitfire | Blaze | Smokey | Forktail |

page 87 • **The Marriage Proposal**

Ruby can take out one pebble and "accidentally" drop it into the river. Then, she'll insist that the pebble was white. Since the other pebble in the purse is black, everyone has to assume that she dropped the white pebble—and she won't have to marry the sneaky prince!

pages 88–89 • **In the Land of Confusion**

**Can you figure out who is a sage and who is a jester?** Xavier and Zachary are sages, and Yale is a jester. Here is the explanation. Xavier must be a sage because if he were a jester, he would lie about it. Since Xavier is a sage, Yale is a jester—his claim that Xavier is lying is untrue. In turn, that means Zachary is telling the truth, so he is a sage.

**Are the boy and girl sages and/or jesters, and what color hair does each one have?** They are both jesters; the boy has red hair and the girl has black hair. If the third statement is true, then either the first or the second statement must be untrue, but that's impossible—either they are both true or both untrue, since one is a boy and the other is a girl. That means the boy is a jester, which means that he is the one who said, "I'm a girl." Therefore, the girl is also a jester, because she said, "I'm a boy."

# PUZZLE ANSWERS

### pages 88–89 • In the Land of Confusion, *continued*

**What can you tell about him or his son?** They are both sages. Let's look at it this way:

1. If the statement is true, then they are either both sages or jesters; since the father is telling the truth, they are both sages.

2. If the statement is false and the father is a jester, his son must be a sage, in which case the statement is not a lie, which means that the father cannot be a jester.

That means the only possible outcome is that they are both sages.

**Immediately, the sages cried: "Imposter! Arrest that man!" Why?** If the King were a sage, he would say the truth, "I'm a sage." If the King were a jester, he would lie and say, "I'm a sage." Since the King said, "I'm a jester," he is neither a sage nor a jester, and is not a citizen of the Land of Confusion.

### pages 90–91 • Read the Numbers

| | | |
|---|---|---|
| CR8TV | 1DRFL | 42N8 |
| 4EVRL8 | I4GOT | 1OSNY1 |
| H8ZW8 | ZBA | W8NC |
| 4CAST | 10Q | SK8R |

### page 92 • Simple Symbols

| | | | | | | |
|---|---|---|---|---|---|---|
| 6 | 2 | 1 | | 2 | 1 | 6 |
| 4 | 1 | 1 | | 1 | 1 | 2 |
| 2 | 1 | 0 | | 1 | 0 | 4 |

### page 92 • Buying Numbers

The customer was buying numbers for her front door.

### page 93 • Can't Find It!

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 2 | 1 | 3 | 1 | 2 | 3 | 5 |
| 2 | 2 | 3 | 3 | 1 | 2 | 1 | 4 | 1 |
| 1 | 2 | 1 | 2 | 5 | 2 | 2 | 1 | 2 |
| 2 | 3 | 1 | 4 | 3 | 1 | 4 | 2 | 5 |
| 1 | 2 | 3 | 4 | 2 | 3 | 3 | 4 | 4 |
| 4 | 1 | 5 | 2 | 1 | 2 | 1 | 3 | 3 |
| 5 | 3 | 4 | 1 | 2 | 4 | 4 | 5 | 5 |
| 1 | 2 | 2 | 1 | 3 | 4 | 5 | 1 | 2 |
| 3 | 1 | 1 | 2 | 4 | 3 | 3 | 2 | 1 |

### page 93 • Found It!

| Students | [illegible] |
|---|---|
| Jasmine | 3 |
| Kellen | 1 |
| Josh | ⊢⊣ |
| Dillon | 5 |
| [illegible] | 13 |

### page 98 • Who Owns That Car?

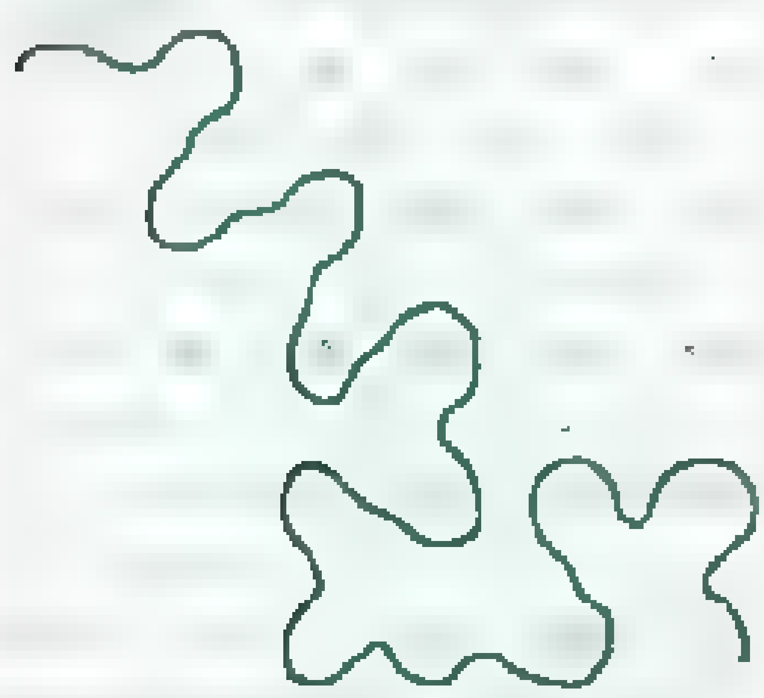
Look at the license plate upside down, and you'll see that the numbers spell out *OLLIE LEE*.

# PUZZLE ANSWERS

page 99 • **Even and Odd**

| x | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | | 2 | | 4 | |
| 2 | 2 | 4 | 6 | 8 | 10 |
| 3 | | 6 | | 12 | |
| 4 | 4 | 8 | 12 | 16 | 20 |
| 5 | | 10 | | 20 | |

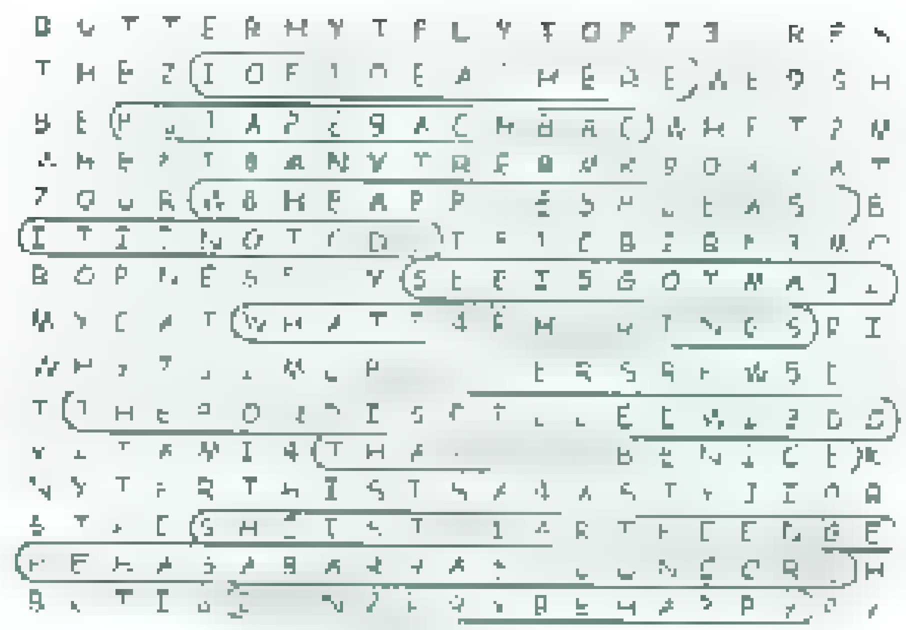
page 100 • **Get Out of Here**



page 101 • **An Average Day**

**One of the averages above is a mode. Which one is it?** Average ice-cream flavor

pages 102–103 • **Hidden Numbers, Etc.**



SAMPLE I often eat here

1 Put a pet in each pen

2 Weigh the apples, please

3 It is even, not odd

4 See if I've got mail

5 What if our phone rings?

6 The pond is filled with reeds

7 That would be nice

8 She is too near the edge!

9 He has a barn in East Concord

10 Don't worry, be happy!

page 106 • **Spellulator**

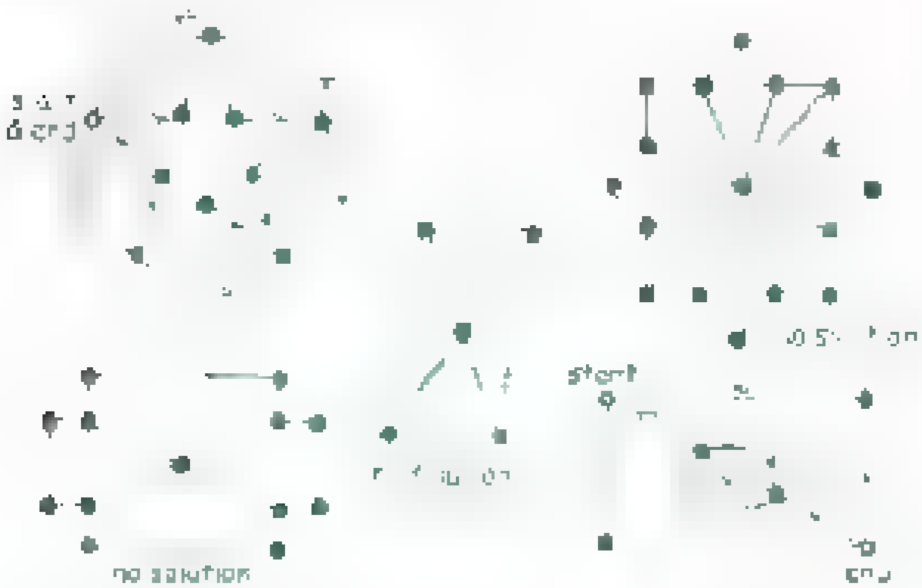
| | |
|---|---|
| 9,645 / 3 = (small, medium, or large) | 3215, SIZE |
| 142 x 5 = (petroleum) | [illegible] |
| 879 x 3 = (what you walk on) | [illegible] |
| 10,000 – 4,662 = (honey makers) | 5338, BEES |
| 50,029 – 15,023 = (barnyard animal) | 35006, GOOSE |
| 206 + 206 + 206 = (the opposite of tiny) | 618, BIG |
| 188,308 + 188,308 = (laugh in a silly way) | 376616, GIGGLE |
| 10 + 13 = (not hard) | 23, EZ |
| 926 x 2 x 2 = (an empty space) | 3704, HOLE |

# PUZZLE ANSWERS

**page 107 • Calling Code**

1. IMA NUTT
2. FRANK N STEIN
3. TEDE BEAR
4. TAFFY PULL
5. DREW A BOAT
6. SUMMERTIME
7. SANTA CLAUS

**page 109 • Practice Solving Networks**




**page 110 • Number Patterns**

**Sequence 1:** + 2 = 14
**Sequence 2:** + 5 = 30
**Sequence 3:** + 4 = 23
**Sequence 4:** 5 + 7 9 + + 13 = 49
**Sequence 5:** × 2 = 96
**Sequence 6:** × 3 = 243
**Sequence 7:** 1, 1, 2, 3, 5, 8, 13; 1 + 1 = 2; 1 + 2 = 3; 2 + 3 = 5; 3 + 5 = 8; 5 + 8 = 13; 8 + 13 = 21
**Sequence 8:** 1, 2, 6, 24, 120; 1 × 2 = 2; 2 × 3 = 6; 6 × 4 = 24; 24 × 5 = 120; 120 × 6 = 720

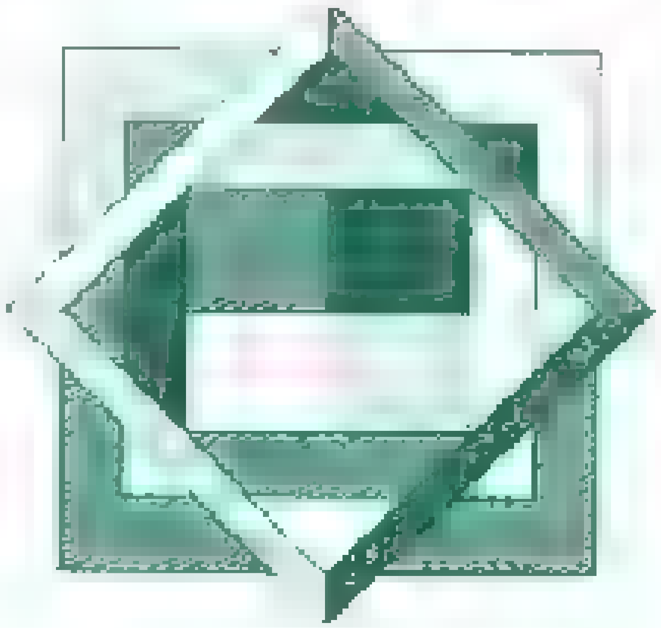
**page 110 • Hopscotch Math**

Turn One = 98 points
Turn Two = 96 points
Turn Three = 94 points
Turn Four = 92 points
Add all four turns together to get 380

**page 111 • Color My World**

**page 111 • Crazy Quilting**

There are several ways you can color this pattern—this is just one way of doing it.



**page 112 • An Unsolved Mystery**

12 = 5 + 7
14 = 11 + 3
16 = 11 + 5
20 = 13 + 7
24 = 11 + 13
34 = 31 + 3
46 = 23 + 23
58 = 53 + 5
60 = 53 + 7

## PUZZLE ANSWERS

**page 113 • Around, and Around, and Around We Go**

**page 115 • The Very Last Cross-Number Puzzle**

| | | | | | |
|---|---|---|---|---|---|
| 1 8 | 2 1 | | | 3 5 | 4 2 |
| 5 7 | 7 | 6 7 | | 7 4 | 9 |
| | | 8 3 | 9 6 | 5 | |
| | 10 1 | 0 | 1 | | |
| 11 2 | 4 | | 12 6 | 13 7 | 14 8 |
| 15 6 | 4 | | | 16 5 | 0 |

**page 114 • That's Just about Right**

Answers will vary. We chose to divide the field of daisies into 24 boxes. There are about 10 daisies in every box. Therefore, we estimate that there are 240 daisies in the field pictured.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